Diversification Return and Leveraged Portfolios

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Diversification Return

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Introduction

- Portfolio rebalancing is a common practice: simple yet beneficial
- Leveraged portfolios are now common
- Previous literatures (*Booth and Fama 1992)* and (*Willenbrock 2011)* on long-only unlevered portfolio
- Personal interests
  - Simple but illustrative model
  - Risk Parity portfolios
  - Mathematical certainty in finance?

Portfolio Rebalance

It generates diversification return

- A simple example – maybe the best example

	Year 1	Year 2
Investment 1	100%	-50%
Investment 2	-50%	100%

- Both investments have 25% arithmetic average return
- Both investments have 0% geometric return after two years but they have perfect negative correlation

Portfolio Rebalance

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It generates diversification return

» Consider a 50/50 portfolio

	Year 1	Rebalance	Year 2
$50	$100	$62.5 (50%)	$31.25
$50	$25	$62.5 (50%)	$125
Total $100	**$125 (25%)**	**$125**	**$156.25 (25%)**

» It generates 25% each year

» But one must **rebalance to the original weights**!

Portfolio Rebalance

Diversification return of long- only portfolios

- Sell winners and buy losers – **mean-reverting strategy**
- The diversification return is always positive

Portfolio Rebalance

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Long- short portfolios

» Consider a 200/100 portfolio

	Year 1	Year 1	Rebalance
$200	$400	114%	$700 (200%)
-$100	-$50	-14%	-$350 (-100%)
Total $100	**$350(250%)**	**100%**	**$350**

» To rebalance, one buys winners and sell losers – **trend-following or momentum strategy**

Portfolio Rebalance

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Diversification return of long- short portfolios

- Buy winners and sell losers
- Is the diversification return negative then?
- Is momentum a losing proposition?

	Year 1	Year 1	Rebalance	Year 2
$200	$400	114%	$700 (200%)	$350
-$100	-$50	-14%	-$350 (-100%)	-$700
Total $100	**$350(250%)**	**100%**	**$350**	**-$300**

Portfolio Rebalance

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Leveraged portfolios

- Hedge fund/Private equity/Real estate/Risk Parity
- Leveraged ETFs: 2X, 3X
- Reverse ETF: -X, -2X

- What happens to diversification returns of these portfolios?
- Does rebalancing sometimes hurt performance?

Diversification Return

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Definition

- Arithmetic average return

$$\mu = \frac{1}{N}\sum_{i=1}^{N} r_i$$

- Geometric average return

$$1 + g = \prod_{i=1}^{N}\left(1 + r_i\right)^{1/N}$$

- Geometric average is less than arithmetic average

$$g \approx \mu - \frac{1}{2}\sigma^2$$

Diversification Return

Definition

- Diversification return definition

$$r_d \equiv g_p - \sum_{i=1}^{N} w_i g_i$$

- Portfolio geometric return in excess of average geometric return
- The whole is bigger than sum of its parts if diversification return is positive

Diversification Return

The key formula

» Diversification return

$$r_d \equiv g_p - \sum_{i=1}^{N} w_i g_i$$

» Diversification return in terms of volatilities (***Willenbrock 2011***)

$$r_d = \mu_p - \frac{1}{2}\sigma_p^2 - \sum_{i=1}^{N} w_i \left(\mu_i - \frac{1}{2}\sigma_i^2 \right)$$

$$\mathbf{r_d} = \frac{\mathbf{1}}{\mathbf{2}} \left(\sum_{\mathbf{i=1}}^{\mathbf{N}} \mathbf{w_i} \boldsymbol{\sigma}_{\mathbf{i}}^{\mathbf{2}} - \boldsymbol{\sigma}_{\mathbf{p}}^{\mathbf{2}} \right)$$

» Diversification return is one half of the weighted average of individual variances minus the portfolio variance

Applications

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Portfolio examples

- Two-asset portfolio with one risky asset
- Leveraged, inverse ETFs
- Portfolio with two risky assets
- Long-only unlevered portfolios
- Leveraged Risk Parity portfolios

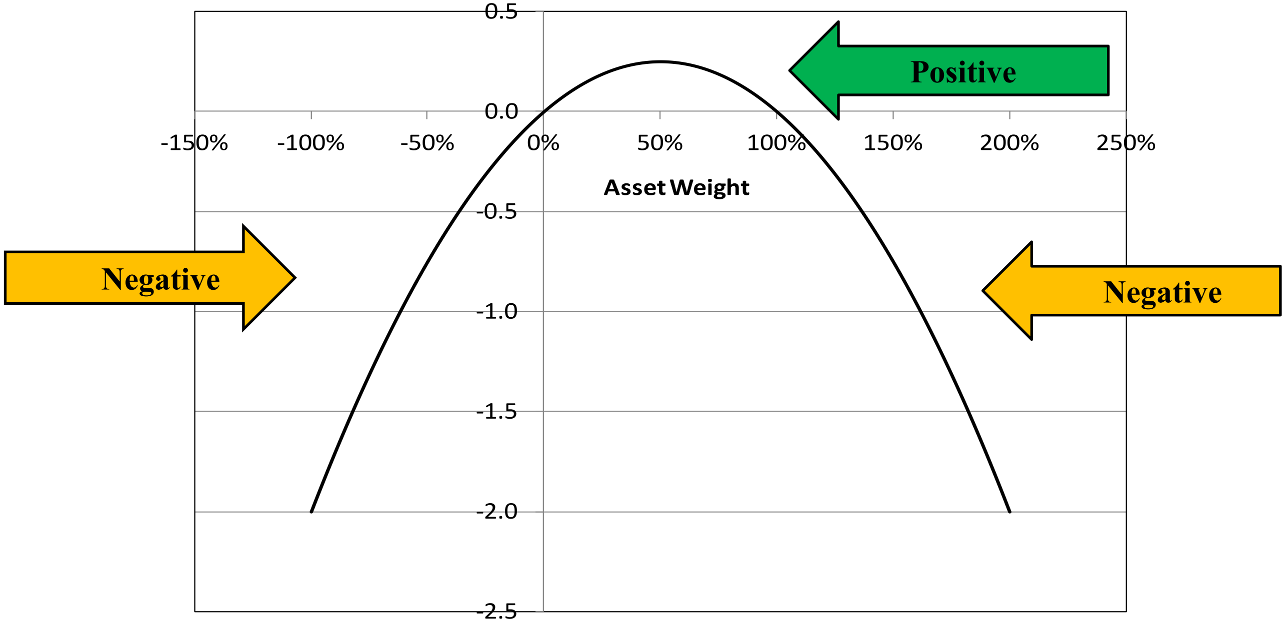
Applications

Portfolio with one risky asset

- One risky asset (w) and risk-free asset

$$r_d = 0.5\left(w - w^2\right)\sigma^2$$

$$\mathbf{r_d} = \frac{1}{2}\left(\sum_{i=1}^{N} w_i \sigma_i^2 - \sigma_p^2\right)$$



Applications

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Ultra long and inverse ETFs

- Match index return multiples **daily**, i.e. 2X, 3X, -X, -2X
  - A single risky asset – cap-weighted indices don't rebalance
- Rebalance on a **daily basis** to the fixed weight
  - Trending-following at the close/Liquidity demanding
- They don't match index return multiple over longer period
- Significant negative diversification returns

Applications

Ultra long and inverse ETFs

» Annual volatility of 23%, daily volatility of 1.45% $23\% / \sqrt{250}$

$$r_d = 0.5\left(w - w^2\right) \cdot (1.45\%)^2$$

» Return slippage

	-3X	-2X	-1X	2X	3X
Daily	-0.1%	-0.1%	0.0%	0.0%	-0.1%
Annual	**-27.6%**	**-14.9%**	**-5.2%**	**-5.2%**	**-14.9%**

Applications

Ultra long and inverse ETFs

- SSO – ultra long, SDS – ultra short
- Flat year in 2011 for the index

	SP 500	SSO (2X)	SDS (-2X)
Multiples	2.11%	4.22%	-4.22%
Slippage		-5.2%	-14.9%
Prediction		-1.24%	-18.51%
Actual		-3.45%	-18.94%

Applications

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Portfolio with two risky assets

$$\mathbf{r_d} = \frac{1}{2}\left(\sum_{i=1}^{N} \mathbf{w_i \sigma_i^2} - \boldsymbol{\sigma}_\mathbf{p}^2\right)$$

» Correlation is now important

$$r_d = \frac{1}{2}\left(w_1\sigma_1^2 + w_2\sigma_2^2 - w_1^2\sigma_1^2 - w_2^2\sigma_2^2 - 2\rho w_1 w_2 \sigma_1 \sigma_1\right)$$

» For long-only portfolios, low correlations leads to high diversification returns

$$r_{d,\max} = \frac{1}{2}\left[w_1\sigma_1^2 + w_2\sigma_2^2 - \left(w_1\sigma_1 - w_2\sigma_2\right)^2\right]$$

$$r_{d,\min} = \frac{1}{2}\left[w_1\sigma_1^2 + w_2\sigma_2^2 - \left(w_1\sigma_1 + w_2\sigma_2\right)^2\right]$$

Applications

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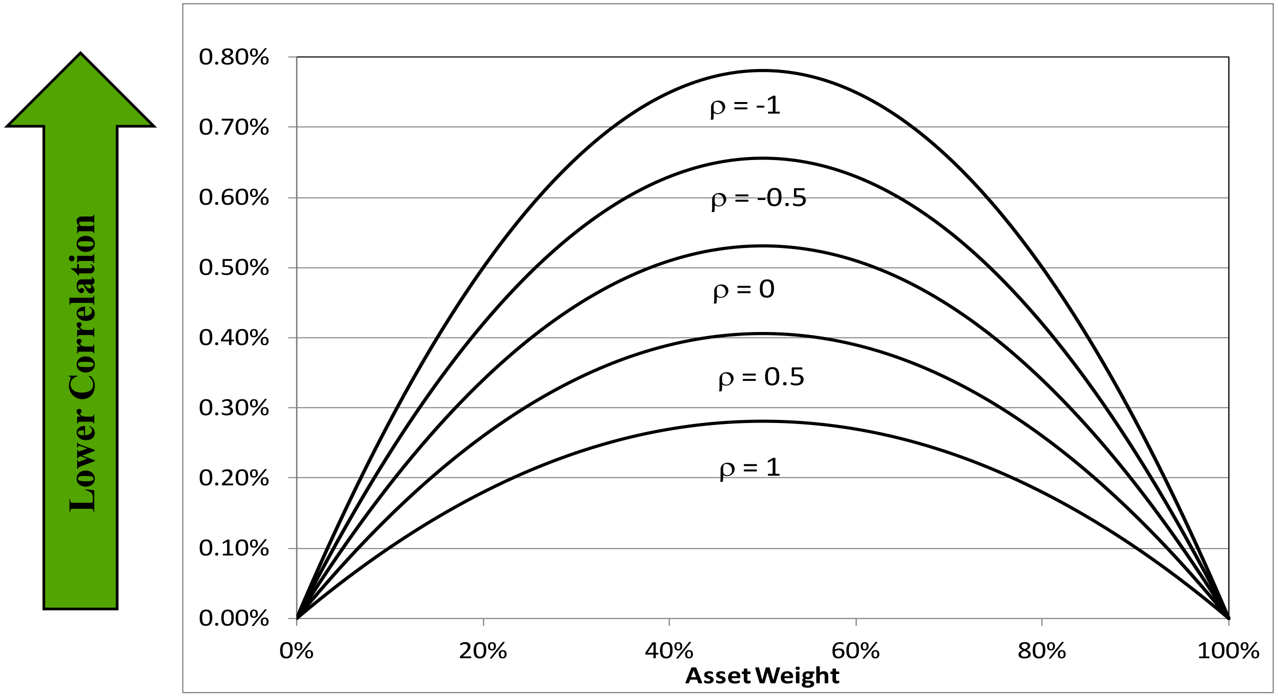
Portfolio with two risky assets

» Asset volatility 5%, and 20%, different correlations



Applications

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Long- only unlevered portfolio

» Diversification return is always non-negative

$$r_d = \frac{1}{2}\left(\sum_{i=1}^{N} w_i\sigma_i^2 - \sigma_p^2\right) \geq \frac{1}{2}\left[\sum_{i=1}^{N} w_i\sigma_i^2 - \left(\sum_{i=1}^{N} w_i\sigma_i\right)^2\right]$$

» It can be proven mathematically that the right-hand side is always non-negative

Applications

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Leveraged long- only portfolios

- What happens when one rebalance a leveraged long-only portfolio?
  - Mean-reverting across different assets (sell winners and buy losers)
  - Trend-following at the portfolio level (de-lever with loss and re-lever with gain) – similar to portfolio insurance
- Its diversification return is a tug of war between the two
  - Leverage increases positive diversification return across assets
  - Leverage produces negative diversification return on top

Applications

Leveraged long- only portfolios

- Rescaled the portfolio to an unlevered portfolio

$$\sum_{i=1}^{N} w_i = L,$$

$$w_i^s = \frac{w_i}{L}, \sum_{i=1}^{N} w_i^s = 1.$$

- The rescaled portfolio has diversification return r_d^s and volatility σ_s
- The diversification return of the levered portfolio is
  - Mutual fund separation theorem

$$r_d = L \cdot r_d^s + \frac{1}{2}\left(L - L^2\right)\sigma_s^2$$

Applications

Leveraged long- only portfolios

» From 60/40 to Risk Parity portfolios

60/40	ρ = -0.5	ρ = 0	ρ = 0.5
Diversification return	0.63%	0.51%	0.39%
Portfolio volatility	11.1%	12.2%	13.1%

20/80	ρ = -0.5	ρ = 0	ρ = 0.5
Diversification return	0.42%	0.34%	0.26%
Portfolio volatility	4.00%	5.66%	6.93%

Risk Parity	ρ = -0.5	ρ = 0	ρ = 0.5
Diversification return	0.77%	0.34%	0.08%
Portfolio volatility	11.1%	12.2%	13.1%
Portfolio leverage	278%	215%	190%

Applications

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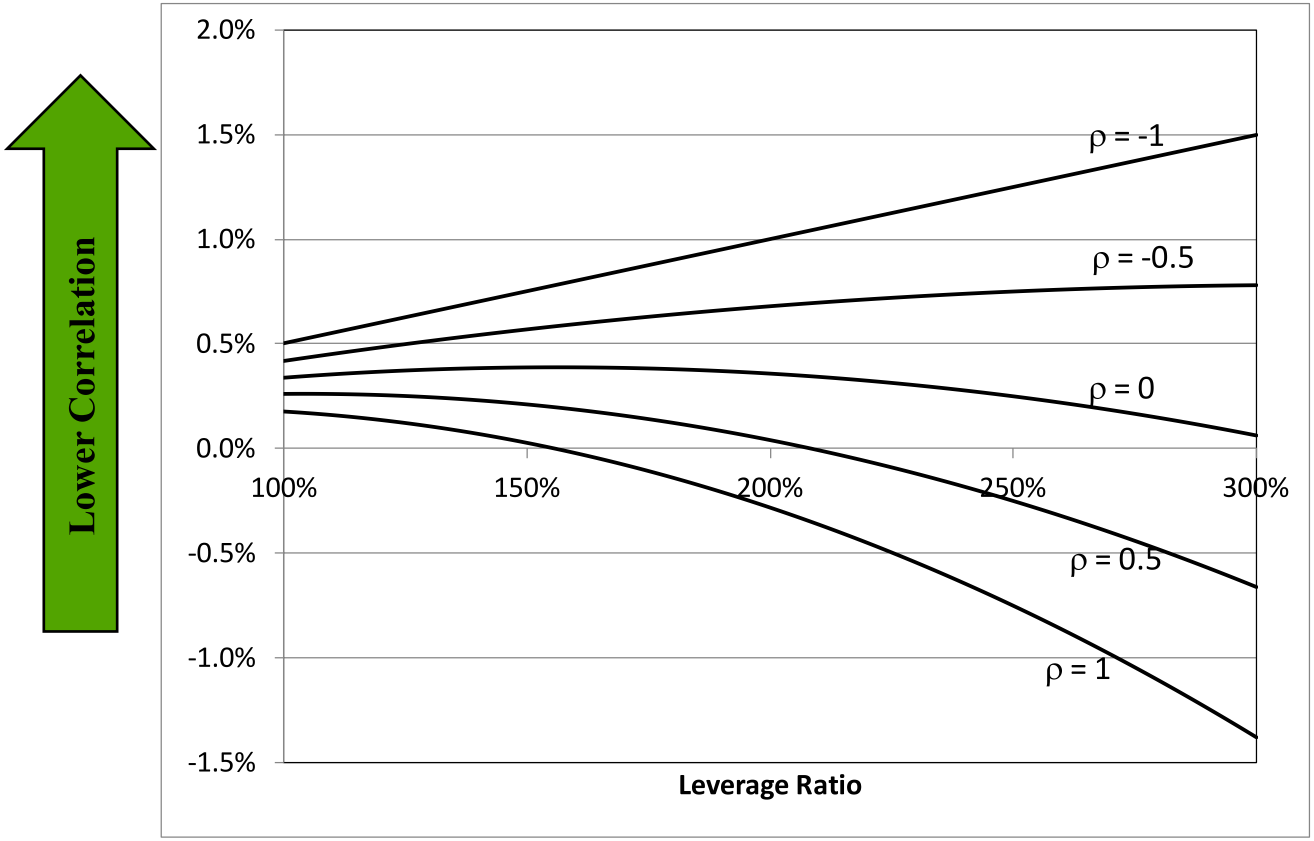
Leveraged long- only portfolios

» Risk Parity portfolios with different correlations



Diversification Return

Summary

- Simple portfolio rebalancing generates diversification returns
- Across assets, rebalancing means mean-reverting
- For leveraged portfolios, top level rebalancing means trend-following
  - Stop-loss is just mechanical
- Long-only unlevered portfolios always have positive diversification returns

Diversification Return

Summary

- Levered and/or inverse ETFs have significant negative diversification returns
- Leveraged long-only portfolios' diversification return consists of two parts
  - Across asset diversification return magnified by leverage - positive
  - Top-down leverage - negative
- The benefit of low correlations: better diversifications, higher diversification returns