Downside Risk

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Motivation

▶ Bearers of downside risk should earn a reward for holding assets that under-perform in bad markets when the preservation of wealth is paramount.

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  - ▶ Going back to the 1950's to the present, does it work as advertised?

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▶ Demonstrate a *tradable, simple to build proxy.*

▶ **Questions:**
  ▶ Going back to the 1950's to the present, does it work as advertised?
  ▶ Is there evidence from an asset pricing framework?

The Literature

Relating stock level returns to return asymmetry.

- **Beta asymmetry:** Bawa & Lindenberg (1977), Ang, Chen & Xing (2005) find that stocks which co-vary strongly with the market when the market declines have high average returns and that the downside risk premium is approximately 6% per annum. *Ex-post* construction.

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- **Tail measures** Bali, Demirtas & Levy (2009) show that VaR dominates expected shortfall and tail risk and on average predicts returns positively.
- **Survey of the Literature** See DiBartolomeo 2007 for a survey of the literature on higher order moments, and their implications for asset pricing models.

The Data

- ▶ **The data,** the HML, SMB and WML portfolios are from Ken French's website. We also downloaded the 10 decile portfolios for Book, Size, Mom and the 38 industry portfolios. The industry portfolios are then reduced to 34 after deleting industries with missing data. Macro data is from the FRED website.

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- **This paper** is most closely tied to Bawa & Lindenberg (1977), and Ang, Chen & Xing 2005. Unlike Ang, whose measure is an ex-post beta measure, the aim is to construct a trade-able portfolio that earns the aforementioned premium.

Downside Risk Motivation

The intuition

Macroeconomic news impacts industries differently.

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- **Chordia & Shivakumar (2000)** find that the profits can be explained by loadings to lagged macroeconomic variables. In our measure, we posit that downside risk exposure proxies for the increased risks associated with the macro driven component to industry returns.

Downside risk as defined in Bawa & Lindenberg (1977)

$$\beta_- = \frac{cov(r_i, r_m \mid r_m < \mu_m)}{var(r_m \mid r_m < \mu_m)} \qquad \beta_{DSR} = \beta_- - \beta \qquad (1)$$

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38 Industry Portfolios

from the Fama French monthly data sets are used (34 after removing those with sparse data.) As suggested by Ang, we use relative beta so that it measures higher expected returns not capured by the CAPM.

Downside Risk Construction

Estimation window

Where Ang et al. use one year of daily data over a forward looking period to construct their measure, we use the most recent two years of monthly history. The split then occurs for the months with market returns less than the two year mean.

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Historical window

extends back to the mid 1950's. The choice is to include only data after T-bills were allowed to vary freely subsequent to the Federal Reserve Accord of 1951, and to include the 1962-2001 period used by Ang, Chen and Xing.

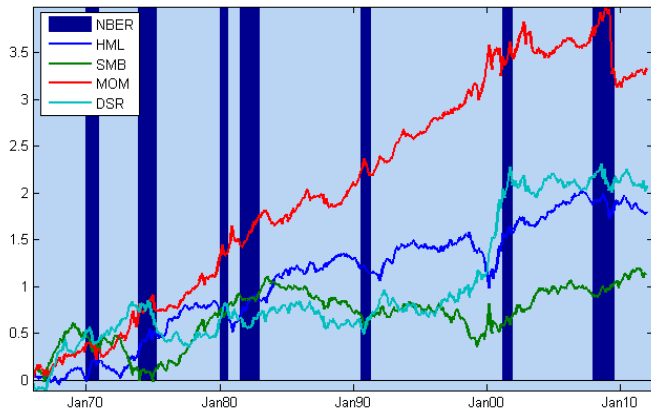Figure: Downside Risk Quintile Portfolios.

Downside Risk

Figure: Downside Risk factor returns with Carhart model.

Market States and Factors: In good times and bad.

	count	mean	t-stat	stdev	skew	kurt
MKT	683.000	0.479	2.826	4.431	-0.535	4.897
HML	682.000	0.346	3.248	2.781	0.019	5.815
SMB	683.000	0.227	1.989	2.985	0.554	9.142
MOM	683.000	0.741	4.714	4.109	-1.450	14.694
IMO	677.000	0.492	3.347	3.827	-1.157	11.583
DSR	677.000	0.315	2.779	2.946	0.201	5.017
Bali	677.000	0.301	2.080	3.764	-0.549	8.100

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SMB	594.000	0.168	1.390	2.950	0.564	10.370
MOM	594.000	0.914	6.051	3.680	-0.439	9.183
IMO	588.000	0.621	4.278	3.517	-0.187	4.057
DSR	588.000	0.411	3.441	2.894	0.349	5.274
Bali	588.000	0.352	2.450	3.488	0.033	4.467

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SMB	89.000	0.620	1.831	3.196	0.460	3.216
MOM	89.000	-0.409	-0.625	6.172	-2.371	13.351
IMO	89.000	-0.354	-0.619	5.401	-2.596	16.716
DSR	89.000	-0.320	-0.938	3.215	-0.398	3.429
Bali	89.000	-0.040	-0.071	5.248	-1.542	10.762

Table: Descriptive statistics for all, up and down market states, from 1955 to 2001. Defined as in Cooper, Gutierrez and Hameed, *Market States and Momentum*, Journal

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Table: Descriptive statistics under Down Market States, from 1955 to 2001.
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Journal of Finance, 2005

Market States and Factors Covariances:

	MKT	HML	SMB	MOM	IMO	DSR	Bali
MKT	19.652	-0.288	0.297	-0.123	-0.141	-0.138	-0.390
HML	-3.557	7.781	-0.218	-0.172	-0.085	0.156	0.060
SMB	3.946	-1.825	8.974	-0.020	-0.075	-0.018	-0.325
MOM	-2.234	-1.969	-0.246	16.852	0.733	0.115	0.372
IMO	-2.401	-0.911	-0.859	11.517	14.647	0.104	0.487
DSR	-1.796	1.281	-0.155	1.395	1.177	8.678	0.134
Bali	-6.511	0.625	-3.666	5.744	7.017	1.484	14.166

	MKT	HML	SMB	MOM	IMO	DSR	Bali
MKT	17.451	-0.393	0.290	0.019	-0.062	-0.130	-0.389
HML	-4.419	7.255	-0.324	-0.199	-0.078	0.131	0.159
SMB	3.589	-2.585	8.777	0.051	-0.038	-0.023	-0.302
MOM	0.292	-1.970	0.554	13.463	0.694	0.121	0.272
IMO	-0.905	-0.739	-0.393	8.950	12.370	0.126	0.414
DSR	-1.574	1.020	-0.198	1.289	1.287	8.374	0.192
Bali	-5.672	1.491	-3.125	3.486	5.080	1.935	12.165

	MKT	HML	SMB	MOM	IMO	DSR	Bali
MKT	34.287	0.107	0.329	-0.511	-0.377	-0.156	-0.390
HML	2.104	11.367	0.296	-0.090	-0.109	0.286	-0.289
SMB	6.151	3.191	10.214	-0.257	-0.208	0.041	-0.427
MOM	-18.465	-1.875	-5.063	38.098	0.829	0.064	0.630
IMO	-11.930	-1.989	-3.588	27.633	29.169	-0.010	0.694
DSR	-2.944	3.098	0.419	1.261	-0.169	10.333	-0.104
Bali	-11.986	-5.111	-7.160	20.415	19.678	-1.756	27.537

Table: Covariances and correlations under all, up and down market states, from 1955 to 2001. Defined as in Cooper, Gutierrez and Hameed, *Market States and Momentum*, Journal of Finance, 2005

const	0.312^{***}	0.310^{***}	0.312^{***}	0.299^{***}
	2.716^{***}	2.825^{***}	2.720^{***}	2.840^{***}
lagLGR		-0.147		-0.339^{**}
		-1.211		-2.325^{**}
DY		-0.031		0.307^{*}
		-0.216		1.716^{*}
AVE3to10		0.126		-0.129
		0.647		-0.597
DIF3to10		0.384^{*}		0.379
		1.929^{*}		1.550
CRED		-0.217		-0.199
		-0.983		-0.753
$const_S$			-0.090	-0.721^{***}
			-0.430	-2.595^{***}
$lagLGR_S$				0.452^{***}
				3.020^{***}
DY_S				-0.592^{*}
				-1.884^{*}
$AVE3to10_S$				0.173
				0.498
$DIF3to10_S$				0.004
				0.023
$CRED_S$				0.284
				0.736
R-sqr	0.000	5.497	0.100	10.567
	NaN	NaN	NaN	NaN
SysRq	-0.000	3.962	0.843	8.857
	NaN	NaN	NaN	NaN

Table: Livingston growth forecast < 0

	(1)	(2)	(3)	(4)
const	0.732^{***}	0.646^{***}	0.732^{***}	0.649^{***}
	5.317^{***}	4.261^{***}	5.338^{***}	4.189^{***}
lagLGR		-0.144		0.019
		-1.233		0.112
DY		0.295		0.499
		0.924		0.952
AVE3to10		0.279		0.095
		1.452		0.302
DIF3to10		0.293		0.281
		1.293		1.053
CRED		-0.416^{*}		-0.403
		-1.670^{*}		-1.451
$const_S$			0.383^{*}	0.570^{*}
			1.756^{*}	1.687^{*}
$lagLGR_S$				-0.108
				-0.515
DY_S				-0.488
				-0.938
$AVE3to10_S$				0.406
				0.877
$DIF3to10_S$				0.253
				0.965
$CRED_S$				0.095
				0.228
R-sqr	-0.000	3.912	1.143	6.880
	NaN	NaN	NaN	NaN
SysRq	-0.000	3.962	0.843	8.857
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Panel regressions using Cooper states

$constant$	0.000	0.000	0.000
$IndustryMOM$	0.162^{***}	0.197^{***}	0.191^{***}
	5.560^{***}	6.095^{***}	5.944^{***}
$DSRisk$	0.085^{***}		0.107^{***}
	3.053^{***}		3.488^{***}
$IndustryMOM_S$		-0.234^{***}	-0.236^{***}
		-3.484^{***}	-3.576^{***}
$DSRisk_S$			-0.188^{***}
			-2.886^{***}
R^2	.0110	.0108	.0142
$Observations$	22,185	22,185	22,185

Table: Lagged market return 6-42 months ago < 0

Asset Pricing Design:
linear beta pricing restriction $H_0 : E\left[R_t\right] = \gamma' \cdot \beta$

The First Pass

$$r_{it} = \alpha_i + \beta_{i1} f_{1t} + \cdots + \beta_{i1} f_{1t} + \epsilon_{it}$$

$$\text{where } \hat{\beta} \doteq \left[\hat{\beta_1}, \cdots, \hat{\beta_K}\right] \text{ risk exposures}$$

$$\text{and } \hat{X} \doteq \left[1_N, \hat{\beta}\right]$$

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The Second Pass

$$R_t = \gamma \hat{X} \text{ where } \gamma \doteq [\gamma_0, \gamma_1, \cdots, \gamma_K]' \text{ risk premia}$$
$$\text{and } \lambda = V_F^{-1} \gamma \text{ price of covariance risk}$$

FFM from 28-Dec-1955 to 28-Dec-2011

R^2: 0.788 (0.159)

H_0: ρ^2=1, 0.2529

H_0: ρ^2=0, 0.0130

Shanken: 0.0317 (0.4269)
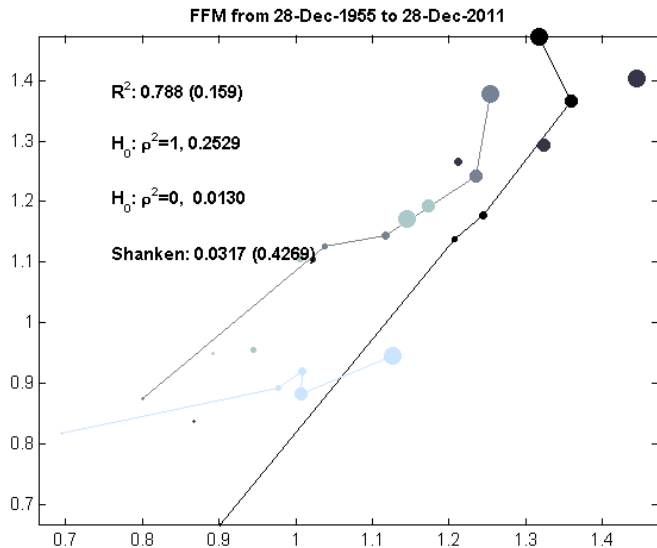
Asset Pricing Design: CAPM Example

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
γ_0	0.642	8.888	8.861	8.674	8.263
γ_{mkt}	0.346	1.813	1.812	1.818	1.812

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
λ_0	0.642	8.888	8.861	8.674	8.263
λ_{mkt}	0.018	1.813	1.803	1.747	1.743

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
γ_0	-0.003	-0.389	-0.386	-0.389	-0.208
γ_{mkt}	0.470	2.747	2.747	2.748	2.764

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
λ_0	-0.003	-0.389	-0.386	-0.389	-0.208
λ_{mkt}	0.024	2.747	2.716	2.556	2.568

Table: CAPM OLS and GLS, 1955 to 2011. Fama-MacBeth which assumes a correctly specified model. Next the Shanken (1992) and Jagannathan and Wang (1998) estimates which still assume correctly specified but account for the EIV estimation error in the betas. Finally the potentially misspecified t-stats of Kan, Ribotti and Shanken 2009.
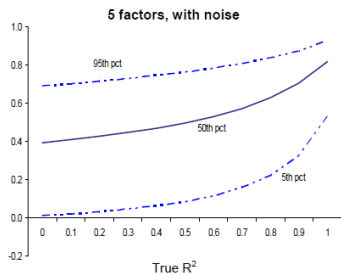
(a) Sample R^2 Confidence Intervals   (b) Adding Industry Portfolios
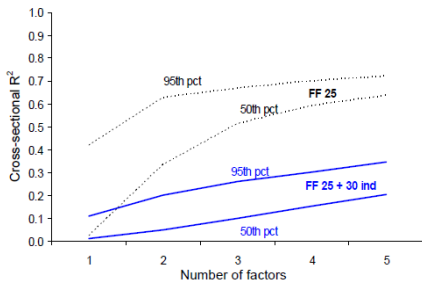
Figure: Low hurdle of traditional pricing designs.
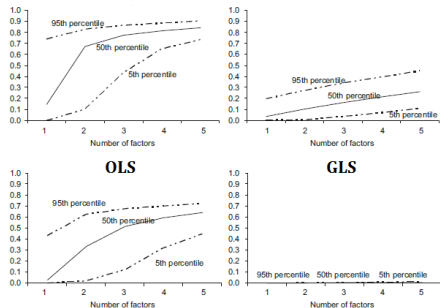
(a) OLS vs GLS
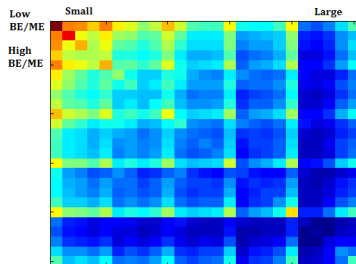
(b) FF25 Covariance Matrix

Figure: Low hurdle of traditional pricing designs.

Lewellen,Nagel & Shanken 2008

Recommendations

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- **Tradable assets on the RHS?** Throw them into the LHS and price them!
- γ_0 Do the slope parameters make sense?

Kan, Robotti & Shanken 2011

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First pass betas are estimated with some error, which introduces an Error in Variables *(EIV)* into the second pass.

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The second pass Fama-MacBeth standard errors are *inconsistent*

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Further information

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KRS contributions

- Misspecification robust standard errors
- Derive the asymptotic distribution of the sample CSR R^2
- Create a test for whether two pricing models have the same population R^2
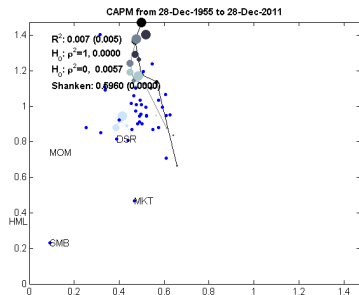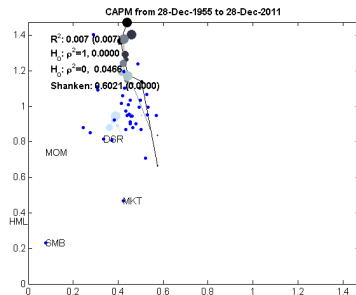
(a) 1955-2011   (b) 1955-2011

Figure: The CAPM GLS and GLS using τ-estimates.

CAPM

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
γ_0	0.642	8.888	8.861	8.674	8.263
γ_{mkt}	0.346	1.813	1.812	1.818	1.812

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
γ_0	-0.003	-0.389	-0.386	-0.389	-0.208
γ_{mkt}	0.470	2.747	2.747	2.748	2.764

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
γ_0	-0.005	-0.780	-0.783	-0.784	-0.414
γ_{mkt}	0.427	2.741	2.326	2.757	1.990

Table: CAPM OLS, GLS, and GLS using τ-estimates. 1955 to 2011.
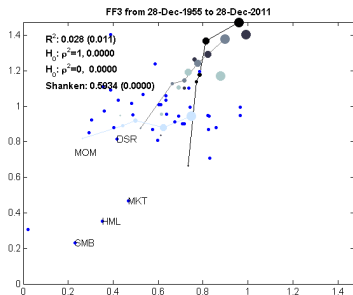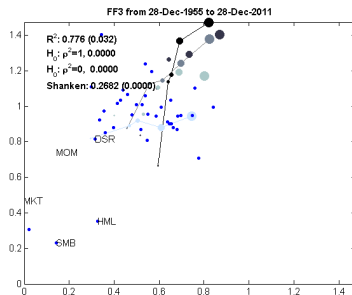
(a) 1955-2011   (b) 1955-2011

Figure: Fama French

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
γ_0	1.542	5.697	5.570	5.713	4.989
γ_{mkt}	-0.644	-1.998	-1.966	-1.995	-1.799
γ_{hml}	0.383	3.451	3.447	3.461	3.456
γ_{smb}	0.177	1.492	1.490	1.498	1.504

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
γ_0	-0.005	-0.814	-0.797	-0.794	-0.433
γ_{mkt}	0.473	2.763	2.763	2.763	2.780
γ_{hml}	0.359	3.323	3.323	3.318	3.287
γ_{smb}	0.236	2.035	2.035	2.037	2.029

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
γ_0	-0.002	-0.258	-0.260	-0.251	-0.197
γ_{mkt}	0.463	28.507	2.695	7.398	7.303
γ_{hml}	0.329	3.605	2.802	3.556	3.187
γ_{smb}	0.144	1.477	1.158	1.463	1.393

Table: Fama French OLS, GLS, and GLS using τ-estimates 1955 to 2011.

Fama French

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
λ_0	1.542	5.697	5.570	5.713	4.989
λ_{mkt}	-0.034	-1.782	-1.739	-1.770	-1.581
λ_{hml}	0.044	2.684	2.610	2.653	2.594
λ_{smb}	0.044	2.845	2.766	2.807	2.728

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
λ_0	-0.005	-0.814	-0.797	-0.794	-0.433
λ_{mkt}	0.031	3.277	3.182	2.895	2.901
λ_{hml}	0.066	4.526	4.365	4.317	4.249
λ_{smb}	0.026	1.906	1.860	1.879	1.863

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
λ_0	-0.002	-0.272	-0.267	-0.266	-0.214
λ_{mkt}	0.032	3.359	3.273	3.017	3.022
λ_{hml}	0.060	4.103	3.983	3.926	3.922
λ_{smb}	0.014	1.040	1.021	1.016	1.012

Table: Fama French OLS, GLS, and GLS using τ-estimates 1955 to 2011.

Fama French DSR
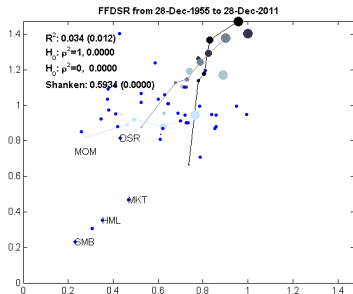
(a) 1955-2001
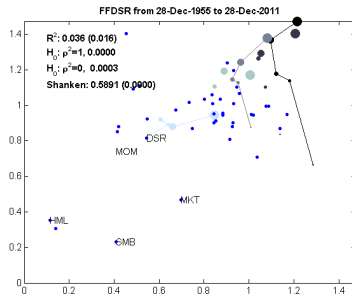
(b) 1955-2011

Figure: Fama French with DSR

Fama French DSR

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
γ_0	-0.006	-0.947	-0.922	-0.918	-0.503
γ_{mkt}	0.473	2.768	2.768	2.768	2.785
γ_{hml}	0.360	3.331	3.331	3.328	3.296
γ_{smb}	0.237	2.042	2.042	2.043	2.036
γ_{dsr}	0.314	2.761	2.760	2.761	2.747

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
γ_0	-0.010	-1.411	-1.407	-1.378	-0.729
γ_{mkt}	0.706	4.381	3.935	4.394	3.355
γ_{hml}	0.124	1.288	1.052	1.250	0.934
γ_{smb}	0.419	4.174	3.323	4.017	3.323
γ_{dsr}	0.149	1.410	1.233	1.401	1.050

Table: Fama French with DSR, GLS, and GLS using τ-estimates 1955 to 2011.

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
λ_0	-0.006	-0.947	-0.922	-0.918	-0.503
λ_{mkt}	0.033	3.541	3.418	3.085	3.087
λ_{hml}	0.061	4.173	4.015	4.009	3.952
λ_{smb}	0.024	1.795	1.744	1.806	1.793
λ_{dsr}	0.035	2.587	2.508	2.512	2.501

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
λ_0	-0.010	-1.481	-1.445	-1.445	-0.801
λ_{mkt}	0.037	3.913	3.778	3.345	3.349
λ_{hml}	0.038	2.614	2.539	2.560	2.524
λ_{smb}	0.038	2.805	2.723	2.879	2.850
λ_{dsr}	0.020	1.488	1.450	1.447	1.440

Table: Fama French OLS, GLS, and GLS using τ-estimates 1955 to 2011.
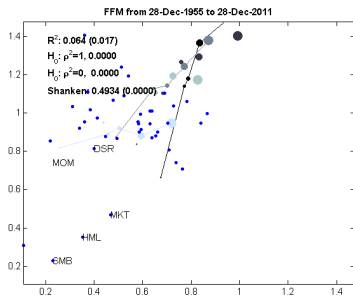
Carhart

(a) 1955-2001
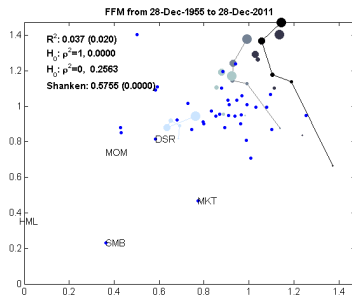
(b) 1955-2011

Figure: Carhart

Carhart

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
γ_0	-0.008	-1.214	-1.156	-1.118	-0.646
γ_{mkt}	0.475	2.778	2.778	2.780	2.794
γ_{hml}	0.361	3.347	3.347	3.352	3.310
γ_{smb}	0.238	2.057	2.057	2.056	2.051
γ_{mom}	0.747	4.709	4.709	4.703	4.682

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
γ_0	-0.022	-1.977	-1.939	-1.900	-1.027
γ_{mkt}	0.797	5.148	4.444	5.179	3.688
γ_{hml}	-0.046	-1.847	-0.411	-0.879	-0.700
γ_{smb}	0.385	3.762	3.078	3.564	2.773
γ_{mom}	0.468	1.720	1.456	1.607	0.383

Table: Fama French GLS, and GLS using τ-estimates 1955 to 2011.

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
λ_0	-0.008	-1.214	-1.156	-1.118	-0.646
λ_{mkt}	0.041	4.331	4.073	3.404	3.414
λ_{hml}	0.087	5.797	5.400	4.911	4.857
λ_{smb}	0.027	2.009	1.908	1.926	1.917
λ_{mom}	0.060	6.211	5.767	4.262	4.216

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
λ_0	-0.022	-3.421	-3.318	-3.272	-1.788
λ_{mkt}	0.044	4.645	4.438	3.751	3.767
λ_{hml}	0.031	2.080	2.011	1.910	1.888
λ_{smb}	0.030	2.236	2.160	2.211	2.187
λ_{mom}	0.038	3.885	3.729	3.190	3.154

Table: Fama French GLS, and GLS using τ-estimates 1955 to 2011.

(a) 1955-2001

(b) 1955-2011

Figure: Carhart with DSR

Carhart with DSR

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
γ_0	-0.008	-1.292	-1.227	-1.198	-0.689
γ_{mkt}	0.476	2.781	2.781	2.783	2.797
γ_{hml}	0.362	3.352	3.352	3.357	3.315
γ_{smb}	0.239	2.062	2.061	2.060	2.056
γ_{mom}	0.748	4.713	4.712	4.708	4.685
γ_{dsr}	0.316	2.780	2.780	2.782	2.766

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
γ_0	-0.010	-1.498	-1.488	-1.449	-0.687
γ_{mkt}	0.702	4.369	3.923	4.376	3.349
γ_{hml}	0.205	2.152	1.744	2.074	1.542
γ_{smb}	0.359	3.547	2.855	3.431	2.655
γ_{mom}	0.137	0.523	0.439	0.487	0.116
γ_{dsr}	0.093	0.881	0.777	0.874	0.667

Table: Fama French GLS, and GLS using τ-estimates 1955 to 2011.

Carhart with DSR

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
λ_0	-0.008	-1.292	-1.227	-1.198	-0.689
λ_{mkt}	0.043	4.471	4.193	3.464	3.472
λ_{hml}	0.083	5.463	5.090	4.666	4.617
λ_{smb}	0.026	1.926	1.825	1.851	1.842
λ_{mom}	0.058	5.929	5.505	4.118	4.078
λ_{dsr}	0.024	1.810	1.716	1.627	1.627

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
λ_0	-0.010	-1.601	-1.557	-1.553	-0.856
λ_{mkt}	0.042	4.429	4.250	3.612	3.622
λ_{hml}	0.057	3.774	3.634	3.494	3.452
λ_{smb}	0.033	2.444	2.367	2.469	2.447
λ_{mom}	0.020	2.071	2.009	1.910	1.902
λ_{dsr}	0.008	0.632	0.614	0.609	0.608

Table: Fama French GLS, and GLS using τ-estimates 1955 to 2011.
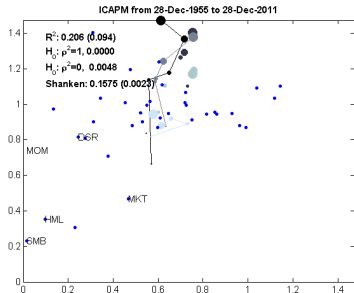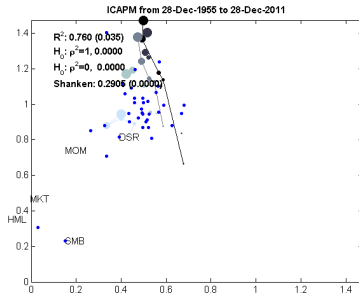
(a) 1955-2001   (b) 1955-2011

Figure: Petkova

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
γ_0	0.003	0.519	0.276	0.266	0.196
γ_{mkt}	0.464	2.712	2.707	2.731	2.742
γ_{dvy}	-0.060	-7.808	-4.612	-4.287	-2.178
γ_{crd}	0.033	3.429	1.955	1.846	0.945
γ_{slp}	-0.045	-1.288	-0.725	-0.692	-0.313
γ_{rfr}	0.075	11.646	6.543	6.172	3.157

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
γ_0	-0.005	-0.334	-0.328	-0.316	-0.181
γ_{mkt}	0.463	27.754	2.692	4.553	4.440
γ_{dvy}	-0.023	-3.225	-2.799	-3.080	-1.603
γ_{crd}	0.025	2.766	2.527	2.794	1.383
γ_{slp}	0.028	0.790	0.728	0.765	0.395
γ_{rfr}	-0.009	-1.602	-1.435	-1.514	-0.815

Table: Petkova GLS, and GLS using τ-estimates 1955 to 2011.

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
λ_0	0.003	0.519	0.276	0.266	0.196
λ_{mkt}	-0.148	-6.266	-3.306	-3.248	-1.741
λ_{dvy}	-12.934	-9.209	-4.813	-4.444	-2.388
λ_{crd}	2.954	3.469	1.840	1.687	0.882
λ_{slp}	1.124	3.618	1.919	1.663	0.833
λ_{rfr}	23.812	13.049	6.705	5.990	3.497

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
λ_0	-0.005	-0.764	-0.722	-0.747	-0.571
λ_{mkt}	-0.020	-0.845	-0.797	-0.873	-0.485
λ_{dvy}	-2.635	-1.876	-1.768	-1.902	-1.003
λ_{crd}	2.196	2.579	2.425	2.617	1.544
λ_{slp}	0.094	0.302	0.285	0.292	0.173
λ_{rfr}	-1.348	-0.739	-0.697	-0.657	-0.397

Table: Petkova GLS, and GLS using τ-estimates 1955 to 2011.
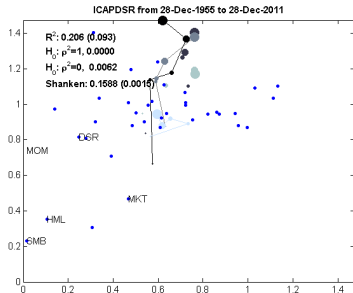
Petkova with DSR
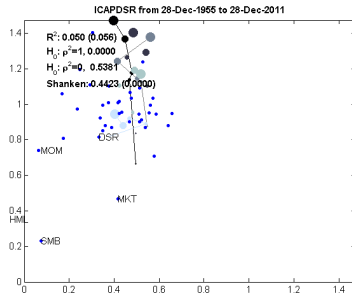
(a) 1955-2001   (b) 1955-2011

Figure: Petkova DSR

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
γ_0	0.003	0.459	0.246	0.238	0.175
γ_{mkt}	0.464	2.714	2.709	2.733	2.745
γ_{dvy}	-0.059	-7.593	-4.495	-4.106	-2.062
γ_{dsr}	0.305	2.680	2.668	2.665	2.647
γ_{crd}	0.031	3.127	1.784	1.728	0.830
γ_{slp}	-0.041	-1.163	-0.657	-0.618	-0.278
γ_{rfr}	0.074	11.570	6.533	6.188	3.124

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
γ_0	0.002	0.147	0.131	0.122	0.042
γ_{mkt}	0.415	2.685	2.165	2.464	1.875
γ_{dvy}	-0.003	-0.422	-0.347	-0.349	-0.113
γ_{dsr}	-0.085	-0.850	-0.673	-0.770	-0.582
γ_{crd}	0.032	3.517	2.905	2.971	0.924
γ_{slp}	0.019	0.557	0.465	0.467	0.127
γ_{rfr}	0.029	5.197	4.281	4.267	1.163

Table: Petkova DSR, GLS, and GLS using τ-estimates 1955 to 2011.

Petkova with DSR

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
λ_0	0.003	0.459	0.246	0.238	0.175
λ_{mkt}	-0.144	-5.979	-3.174	-3.041	-1.615
λ_{dvy}	-12.741	-8.917	-4.692	-4.251	-2.249
λ_{dsr}	0.010	0.736	0.393	0.384	0.313
λ_{crd}	2.748	3.066	1.637	1.539	0.754
λ_{slp}	1.160	3.688	1.967	1.682	0.838
λ_{rfr}	23.749	13.000	6.717	6.035	3.483

	coeff	t_{FM}	t_{Sh}	t_{EIV}	t_{ms}
λ_0	0.002	0.338	0.286	0.276	0.149
λ_{mkt}	0.028	1.156	0.977	0.883	0.314
λ_{dvy}	-0.441	-0.308	-0.261	-0.234	-0.079
λ_{dsr}	-0.019	-1.345	-1.136	-1.099	-0.681
λ_{crd}	2.973	3.317	2.790	2.920	0.933
λ_{slp}	0.699	2.222	1.875	1.730	0.558
λ_{rfr}	9.281	5.080	4.240	4.235	1.439

Table: Petkova GLS, and GLS using τ-estimates 1955 to 2011.

Differences in sample R^2

	FF3	FFM	ICAPM	FFDSR	FFMDSR	ICAPDSR
CAPM	-0.219	-0.365	-0.235	-0.225	-0.371	-0.236
	0.014	0.001	0.226	0.010	0.001	0.263
FF3		-0.146	-0.015	-0.005	-0.152	-0.016
		0.000	0.008	0.273	0.000	0.012
FFM			0.131	0.141	-0.006	0.130
			0.239	0.000	0.251	0.129
ICAPM				0.010	-0.136	-0.001
				0.009	0.228	0.815
FFDSR					-0.146	-0.011
					0.000	0.013
FFMDSR						0.135
						0.121

Table: Differences using GLS.

Differences in sample R^2

	FF3	FFM	ICAPM	FFDSR	FFMDSR	ICAPDSR
CAPM	-0.100	-0.189	-0.140	-0.102	-0.189	-0.144
	0.203	0.073	0.207	0.254	0.089	0.257
FF3		-0.088	-0.040	-0.002	-0.088	-0.043
		0.014	0.199	0.711	0.033	0.207
FFM			0.048	0.087	-0.000	0.045
			0.904	0.006	0.930	0.902
ICAPM				0.038	-0.048	-0.003
				0.164	0.985	0.609
FFDSR					-0.087	-0.042
					0.012	0.166
FFMDSR						0.045
						0.982

Table: Differences using GLS estimates.

Differences in sample R^2

	FF3	FFM	ICAPM	FFDSR	FFMDSR	ICAPDSR
CAPM	-0.100	-0.189	-0.140	-0.102	-0.189	-0.144
	0.203	0.073	0.207	0.254	0.089	0.257
FF3		-0.088	-0.040	-0.002	-0.088	-0.043
		0.014	0.199	0.711	0.033	0.207
FFM			0.048	0.087	-0.000	0.045
			0.904	0.006	0.930	0.902
ICAPM				0.038	-0.048	-0.003
				0.164	0.985	0.609
FFDSR					-0.087	-0.042
					0.012	0.166
FFMDSR						0.045
						0.982

Table: Differences using GLS robust τ-estimates.

Model Comparisons

	CAPM	FF3	FFM	ICAPM	FFDSR	FFMDSR	ICAPDSR
$\hat{\rho}^2$	0.007	0.028	0.064	0.206	0.034	0.067	0.206
$p(\rho^2 = 1)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$p(\rho^2 = 0)$	0.006	0.000	0.000	0.005	0.000	0.000	0.006
$se(\hat{\rho}^2)$	0.005	0.011	0.017	0.094	0.012	0.018	0.093
$\hat{Q}_c$	0.596	0.593	0.493	0.157	0.593	0.490	0.159
$p_1(Q_c = 0)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$p_2(Q_c = 0)$	0.000	0.000	0.000	0.002	0.000	0.000	0.002
No. of pars	2.000	4.000	5.000	6.000	5.000	6.000	7.000

Table: Differences using GLS estimates.

Model Comparisons

	CAPM	FF3	FFM	ICAPM	FFDSR	FFMDSR	ICAPDSR
$\hat{\rho}^2$	0.007	0.776	0.037	0.760	0.036	0.038	0.050
$p(\rho^2 = 1)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$p(\rho^2 = 0)$	0.047	0.000	0.256	0.000	0.000	0.200	0.538
$se(\hat{\rho}^2)$	0.007	0.032	0.020	0.035	0.016	0.016	0.056
$\hat{Q}_c$	0.602	0.268	0.575	0.291	0.589	0.588	0.442
$p_1(Q_c = 0)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$p_2(Q_c = 0)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000
No. of pars	2.000	4.000	5.000	6.000	5.000	6.000	7.000

Table: Differences using GLS robust τ-estimates.

Conclusions and musings