Why Factor Risk Models Often Fail Active Quantitative Managers? The Completeness Conflict

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It is routine among quantitatively-oriented equity portfolio managers to use some form of linear factor risk model to measure the expected variation of portfolio returns. A wide variety of linear factor risk models are available from several vendors (including Northfield, BARRA, Vestek, APT Associates, etc.). Typically, the active variation from some benchmark index is often greatest concern as a metric. While the theoretical and empirical benefits of linear factor models for measuring portfolio risks are very strong, it must be admitted that from time to time, there appears to be a breakdown in the predictive ability of these models. Such breakdowns often result in dispersion of ex-post portfolio returns which is far greater than predicted by such model. It is our hypothesis that such failures arise from a simple mathematical conflict when linear factor risk models are combined with many quantitative security selection strategies. With appropriate handling of this conflict, such failures can be avoided.

Linear factor risk models express the expected covariance matrix of security returns in the form of a factor covariance matrix to which each security is exposed and a security-specific portion. The usual mathematical formulation is:

(1) $V_p = \text{SUM} \left[_{i=1 \text{ to } n} \text{SUM} \left[_{j=1 \text{ to } n} E_{pi} E_{pj} S_i S_j R_{ij} \right]\right] + \text{SUM} \left[_{k=1 \text{ to } m} W_k^2 S_k^2\right]$

(2) $E_{pi} = \text{SUM} \left[_{k=1 \text{ to } m} W_k * B_{ki}\right]$

Where V_p = variance of portfolio return
     n = number of factors in the risk model
     m = number of securities in the portfolio
     E_{pi} = exposure of the portfolio to factor i
     S_i = standard deviation returns attributed to factor i
     R_{ij} = correlation between returns to factor i and factor j
     W_k = weight of security k in the portfolio
     S_k = standard deviation of security specific returns for security k
     B_{ki} = beta of security k to factor I

One of the key assumptions of this formulation regards the item called security specific returns. As is implied the terminology, it is assumed that such returns indeed specific to a given security and hence are uncorrelated between each possible pairing of securities within the portfolio. We are assuming that the factors specified in the model are sufficient to explain all covariance among the securities. In this sense, we presume that the factor

specification is *complete*. We should note that completeness is used in a slightly different context than in most academic literature, which uses completeness to refer to the number of factors necessary to describe the equilibrium relationship between risk and return. In our case, we make no assumptions about equilibrium but rather about the exhibition of no pairwise correlation in residual returns.

We can easily relax this completeness assumption. Let's assume that our factors do not fully specify all of the covariance among securities. In this case, we have:

(3) $V_p = \text{SUM} [_{i= 1 \text{ to } n} \text{SUM} [_{j= 1 \text{ to } n} E_{pi} E_{pj} S_i S_j R_{ij}]] + \text{SUM} [_{k= 1 \text{ to } m} \text{SUM} [_{l= 1 \text{ to } m} W_k W_l Z_k Z_l P_{kl}]]$

Where Z_k = the standard deviation of residual return for security k
P_{kl} = the correlation between residual returns of security k and security l

It is easy to see that for the special case where $P_{kl} = 0$ for all k not equal to l, then $S_k = Z_k$ and equations (1) and (3) are equivalent.

Now let us turn to the issue of security selection strategies when combined with a linear factor risk model. Active managers seek to produce returns greater than their benchmark index in a variety of ways. One way to do this is to assume that the mean expected return for a factor in the risk model will be positive or negative. While our risk model specifies the expected standard deviation of factor returns, we are still able to make any assumptions about the expected means of factor returns without violating any assumptions of equation (1). So managers can construct "factor bets" by emphasizing exposure to a particular factor without any conflict with risk model assumptions.

Another way to manage actively is to select each security we choose to hold on the basis that each chosen security has some particular attribute that makes the expectation of its stock specific return will be positive. For example, we might belief that one security will produce superior returns due to excellent products while another security might produce superior returns due to an unusually strong balance sheet and a third would produce superior returns due to a recent acquisition of a competitor. As long as the reason for the expectation of superior performance in each case is independent, no correlation in the security specific returns is implied and equation (1) still holds.

However, among quantitatively oriented equity managers the preponderance select securities by means of a "multiple-factor model". Unlike a risk model where the factors are chosen to best explain the covariance of securities, the factors in these selection models are meant to identify securities which will provide superior mean returns. If the factors in the selection model are the same as in the risk model (or are highly correlated proxies), the assumptions of equation (1) still hold.

If the factors in the selection model are different from the factors in the risk model we have a very probable conflict. The factors in the selection model are characteristics shared in common by multiple securities which we believe will cause those securities to behave alike in that they are expected to provide us with superior returns in the future. In short, using a factor selection model (with different factors than the risk model) can only produce superior returns if residual returns exhibit covariance. In essence, we must now assume that that our risk model is not complete, therefore equation (1) does not hold and that equation (3) is the proper representation of the problem.

Mathematically, it would be possible to have different selection factors and have equation (1) still hold if we can demonstrate that the returns associated with the selection factors are perfectly constant over time. For example, one might build a model of the mean residual return as a function of the values of security selection variables at the start of the sample period. Unfortunately, time series correlation in the residuals will be induced if the explanatory power (information coefficient) of the predictive variable is not constant over time . To our knowledge, no financial researcher has ever claimed to have found such a constant return factor (with the possible exception of very short investment time horizons.)

For practitioners, the problem is hopefully clear: you can't have it both ways. Either equation (1) is appropriate or equation (3) is appropriate. Yet, it is common practice to measure risk using the assumptions of uncorrelated residuals [equation (1)], while using stock selection strategies that can only work if equation (1) does not hold (no factor bets). Since active mangers will be concentrating their portfolios in those securities expected to act alike in providing superior returns, the average residual covariance will be positive. The second term of equation (3) will have a larger positive magnitude than the second term of equation (1). Hence, if we use a risk model which assumes uncorrelated residuals we will have a downward biased estimate of the risks not identified by the factors of our risk model.

There are several things that portfolio managers can do to mitigate this problem. First, a good dose of humility is in order. The skill of portfolio managers is routinely measured using the ratio of excess returns (relative to benchmark) divided by the standard deviation of excess returns (square root of the variance as calculated in our equations, also known as tracking error). This ratio is often called the information ratio. By examination of manager return databases, it is easy to see that successful managers (top quartile) average ex-post information ratios is on the order of one half. If the expected information ratio (net of costs) for a portfolio is far greater than one-half, it is quite probable that a substantial modeling conflict exists: either the expected excess returns or the expected risk estimates are just too good to be true.

Another useful approach is to simply assume that the risk model's estimate of non-factor risk will be downward biased. By reducing our willingness to take non-factor risks, we can control non-factor risk to have a small magnitude, hence any downward bias in the

estimate of non-factor risk will also have small magnitude. Unfortunately, some practitioners take the opposite view, trying to be tolerant of non-factor risk so as to "allow room" for their security selection strategies to operate.

One should also be particularly careful with multi-factor selection strategies where some but not all of the selection factors are in common with the risk model. In such cases, the risk model will tend to neutralize exposure to the risk factors, which will lead to the selection of a security set which are weak on the selection factors which are in common with the risk model, while having extremely high exposure to the selection factors which are presumed independent of the risk model. As such, it is easy for a multi-factor selection strategy to become dependent on a single component, defeating the purpose of the multi-factor construct.

It should be noted that being particularly averse to non-factor risks will also tend to reduce our factor bets as well. In order to reduce non-factor risk we must tend to hold the same securities as the benchmark in similar proportions. In such a portfolio, the factor bets will necessarily be small. It is not possible to have the same stocks in the same weights but have different factor exposures than the benchmark. However, the converse is not true, for it is very easy to construct portfolios with the same factor exposures as the benchmark with entirely different securities. This asymmetry should cause practitioners to exhibit different tolerances to factor related and security specific risks.