Non-Parametric Methods for Asset Allocation in Private Wealth

Forthcoming in Quant Methods for High Net Worth Investors
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Introduction

- The Analytic Hierarchy Process is a non-parametric technique that is particularly well suited for making complex decisions in a robust fashion.
- One class of such decisions is the question of asset allocation for high net-worth households. There are many considerations that are difficult to reduce to the familiar two-parameter Markowitz problem.
- There may be issues of a non-economic nature arising from the attitudes and experiences of the investor, as well as questions of legal aspects of suitability, liquidity, taxation and inter-generational transfers.
- The AHP is especially useful as an adjunct to traditional mean-variance methods. It can effectively address the popular criticism of mean-variance methods with respect to a severe lack of robustness of optimal solutions due to estimation error.

First Two Topics in the Presentation

- Review of the Analytic Hierarchy Process
  - How has the AHP has been adapted for use in decision making with respect to investor asset allocation and investment selection?
  - AHP allows for "expert" knowledge of the various complexities of the problem to be conveniently and consistently incorporated into the solutions, while remaining generally consistent with the solutions offered by mean-variance methods.

- Next we will consider how AHP can be used to improve the robustness of optimal solutions of the traditional mean-variance method.
  - We will argue that expert knowledge encapsulated in an AHP approach to asset allocation is comparable in concept but more comprehensive than other means of dealing with estimation error.

Third Topic in the Presentation

- The third aspect of the study will utilize the AHP in combination with conventional mean-variance parameter estimation to obtain inferences as to the degree of risk aversion of the investor.
  - While the Markowitz efficient frontier process computes a range of efficient portfolios based on return and risk expectations, there has relatively little research into how investors should define their own risk aversion so as to be able to choose which portfolio on the efficient frontier they should hold.
  - This is particularly important as financial market conditions vary over time. As markets move, investors and their advisors often struggle to come to decisions about how to revise the portfolio to meet the new circumstances.
  - Use of AHP to formally estimate risk aversion can help because the investor can usefully distinguish between changes in portfolio composition which might arise from changes in capital market expectations and changes in portfolio composition which might arise if the investor explicitly chooses to change their level of risk aversion.

Last Topic in the Presentation

- Our final topic will be to consider how AHP can be juxtaposed with the "Discretionary Wealth Hypothesis" of Wilcox (2003).
  - Wilcox proposes an analytical solution to the question of what an investor's mean-variance risk aversion should be.
  - The optimal degree of risk aversion at any moment in time is derived from a "life balance sheet" representation of the investor's assets and liabilities, inclusive of the present values of expected future savings on the asset side, and expected future consumptions expenditure
  - Very wealthy households have assets that often exceed their liabilities by orders of magnitude. We propose to use AHP and the DWH in concert to instead estimate a "shadow" liability component to the investor's life balance sheet.
  - The shadow liability is the present value of a perpetual income stream to be supported by the household wealth, similar in concept to an endowment fund to be provided income to future generations. This concept can be reversed to provide *advice on savings rates to less affluent households*.

The Analytic Hierarchy Process

- The Analytic Hierarchy Process is one of the most widely used mathematical applications in decision theory.
  - Originally developed by Thomas Saaty in the 1970s, the AHP has been used for an almost innumerable set of applications from ranking the importance of historic buildings for the purpose of budgeting funds for restoration to determining where industrial firms should locate off-shore manufacturing plants for optimal long-term profitability.
  - Under certain assumptions it is mathematically demonstrable that the AHP produces optimal decision outcomes for a wide variety of problems, particularly those involving multiple, potential conflicting criteria or contributing opinions.
  - An extensive discussion of the numerical aspects of the AHP can be found in Saaty (1980).

AHP Basics

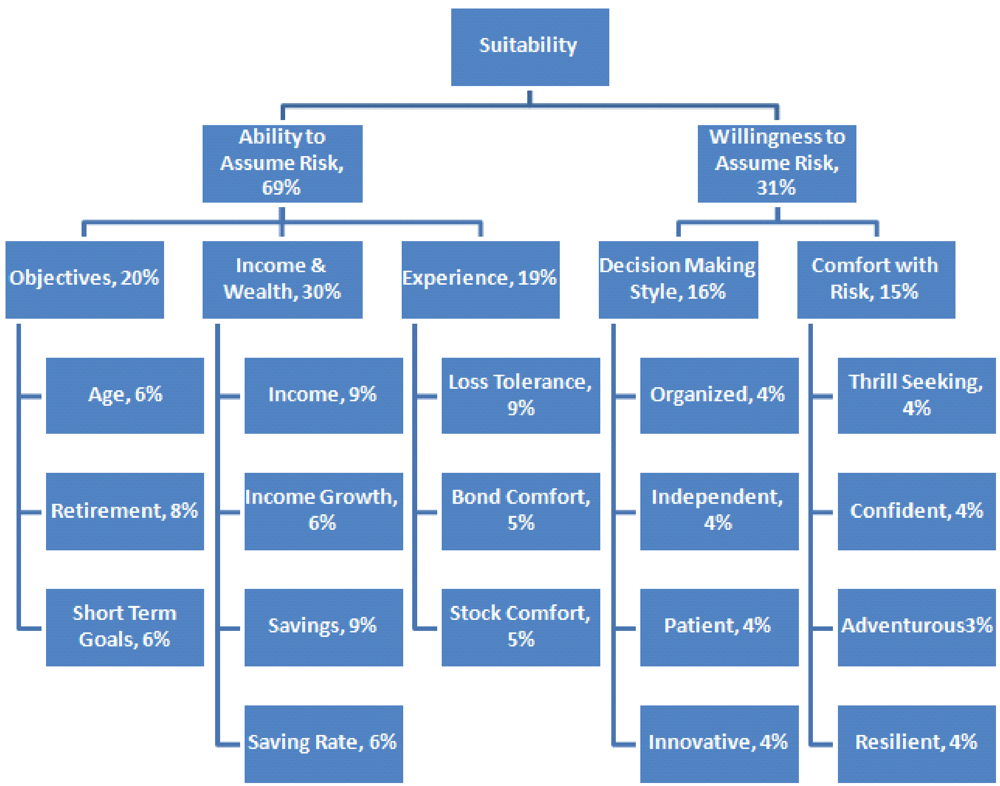
- The basic idea of AHP is to formulate multiple decision criteria in the form of questions, with each question having multiple discrete answers (e.g. a typical multiple choice test).
- For each possible question response (such as an asset class), every solution choice is subjectively evaluated in a pair-wise comparison.
  - For example, we might say that in the absence of other information, that an individual between 25 and 35 years of age is three times better off investing in equities than holding cash.
  - Similarly, we might believe that in the absence of other information that bonds are four times as good an investment as small cap stocks for an individual living on a modest retirement income.
  - By rating all possible pair-wise comparisons possible given the set of questions, answers and solution choices we form a set of numerical matrices that describes all of the relevant information and preferences
  - The eigenvectors of these matrices have special properties that identify optimal solutions.

Asset Allocation and Suitability

- Given the complex objectives of real world investors, AHP would seem to be a natural complement to Markowitz (1952).
  - AHP first proposed as an alternative asset allocation method for investors in Khasari, Kamath and Grieves (1989).
  - The ability of AHP to address investor objectives other than simple return and risk was explored in Bolster, Janjigian and Trahan (1995). This paper first addressed the issue of how to incorporate the legal concept of “suitability” into the portfolio allocation process.
  - Investment services are subject to substantial “suitability” regulation in most countries. The conceptual basis of most of these regulations arises from the “prudent man rule” (Justice Samuel Putnam in 1830)
  - Some countries including the USA (e.g. Dodd-Frank Act 2010) have tightened the regulatory framework for financial institutions dealing with individual investors to create a higher degree of fiduciary responsibility.
  - Saraoglu and Detzler (2002) uses AHP to match retail investor preferences for portfolio management experience and fee levels to mutual funds.

An AHP Illustration

Figure 1



Reproduced from Bolster, P. and S. Warrick, "Matching Investors with Suitable, Optimal and Investable Portfolios", Journal of Wealth Management, 2008.

Bolster and Warrick (2008)

- The most important paper in the AHP finance literature is that of Bolster and Warrick (2008).
  - In this paper which is focused on private wealth investors, AHP and traditional mean-variance are first compared to illustrate the fact that "suitable" portfolios formed using AHP methods are often "close enough" to mean-variance efficient under the criteria established in Jobson (1991).
  - This work then uses process of Black and Litterman (1992) to conform the set of AHP portfolios and the set of portfolios on the adjusted mean-variance efficient frontier until the differences are no longer statistically significant under Jobson test.
  - The paper also combines AHP and mean-variance optimization to conform the portfolios created to real world constraints encountered by retail investors. Such constraints would include minimum dollar amounts that can be invested in particular funds, or the unwillingness of an investor to hold more than a selected number asset classes to be chosen out of a larger set.

Familiar History of Estimation Error in Optimization

- The original formulation of mean-variance optimization from Markowitz (1952, 1959) *assumes* that the return characteristics of the included assets are known only as distributions, but that the parameters of these distributions (mean, standard deviation and correlation) are known with exactness.
  - In the real world, these statistical parameters are available only *as estimates of a highly uncertain future*. The casual substitution of estimated parameters for known parameters has led to widespread criticism of mean-variance optimal solutions as lacking robustness, wherein small changes in parameters lead to disproportionately large changes in asset allocation solutions.
  - Papers by Barry (1974) and Michaud (1989) delve into the mathematical roots of the instability of optimization solutions. In Jorion (1992) and Broadie (1993) Monte-Carlo simulation methods are used to estimate the magnitude of potential "wrongness" of purportedly mean-variance optimal solutions when parameters are estimated rather than known.

Solving the Lack of Robustness

- Combining AHP methods into traditional mean-variance optimization can resolve solution instability more efficiently than other procedures
  - The goals are the same: first to ensure that asset allocation portfolios are well diversified without undue concentration in any one asset class, second to promote smooth transitions in asset weights as we increase the risk tolerance of the investor, and third to reduce sharp changes in asset weights as the result of small changes in return and risk assumptions (i.e. adding to robustness).
- Two Alternative Approaches
  - Add additional terms to objective function, as in Goldfarb and Inyegar(2003) and Haldorsson and Tutuncu (2003).
  - The second approach is to include Monte-Carlo simulation processes into the optimization process itself. Bey, Burgess and Cook (1990). A particular variant (parametric resampling) was described and patented by Michaud (1998). Gold (1995) extends the method of Bey, et. al. to the special case of illiquid assets such as real estate.

Bayesian Methods

- The most popular approach is to adjust to the various input parameters to the optimization using Bayesian statistics.
  - In essence, we will form a “common sense” solution (i.e. “a prior”) to the asset allocation problem and blend this common sense solution with the Markowitz optimal. An early example of this framing of the problem is Jorion (1986).
  - The most widely used methodology for dealing with estimation error in a Bayesian fashion is that of Black and Litterman (1992) which begins with the assumption that the world wealth portfolio is efficient and then allows the investor to impose views of the attractiveness of different markets.
  - Essentially we run the optimization problem in reverse to find “implied” expected returns that would make the world wealth portfolio efficient. We can then blend these implied returns with our own estimates.

More on Black Litterman

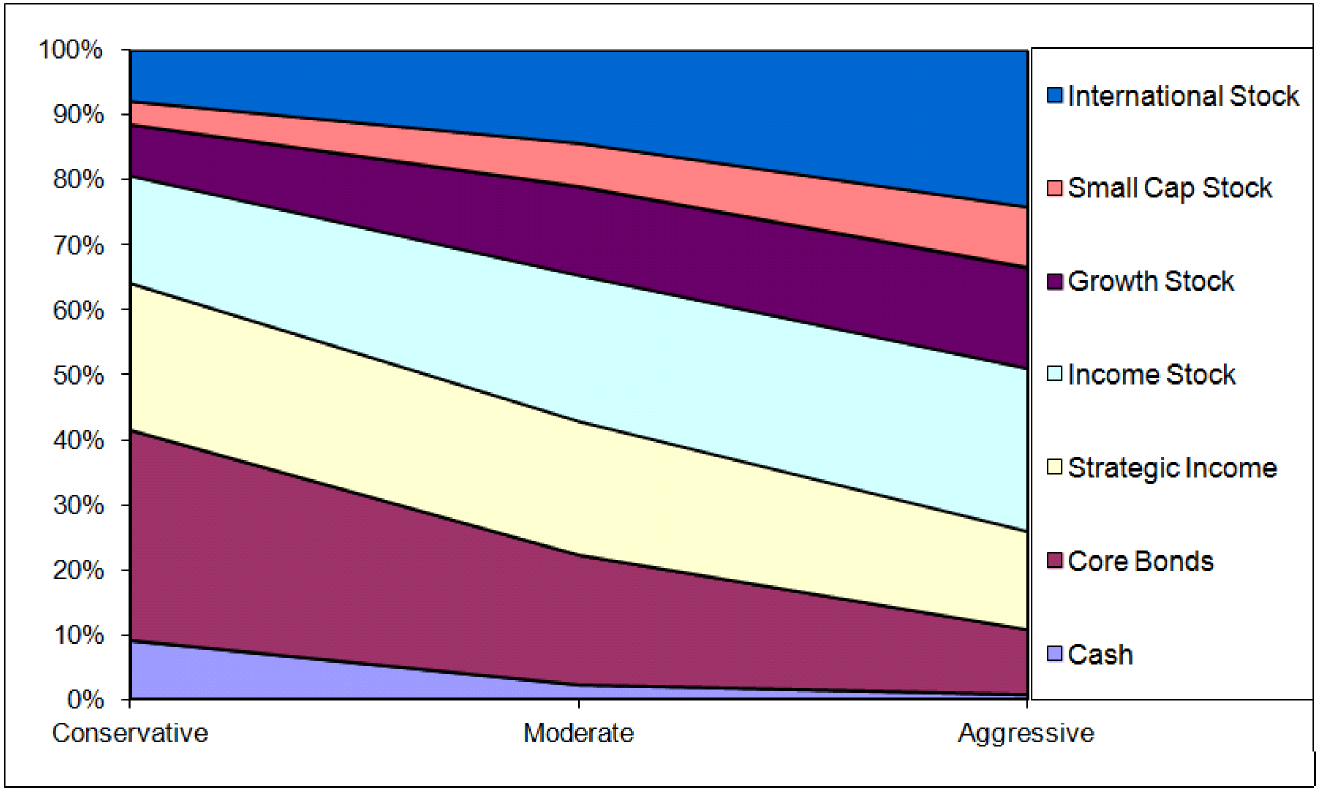
- Alternatively, we can specify an entire portfolio that we believe is appropriate for the investor and by the same inference process find the returns that make our preferred portfolio mean-variance efficient.
  - This second set of implied returns would then be blended with the returns implied from the efficient market assumption to form the inputs for the actual optimization problem. A good discussion of the practicalities of BL can be found in Idzorek (2003).
- Recent criticism of the Black-Litterman (BL) method has come forward in Michaud, Esch and Michaud (2013).
  - The first key criticism is that the BL method requires that the covariance of the assets is assumed to be known, not estimated.
  - A second criticism is that BL assumes the existence of a known global "market portfolio", and there is widespread disagreement among investors as to the composition of such a portfolio.
  - Another concern is that it is possible for the BL methodology to create optimal portfolios with short positions (negative weights), which are not legally investable by many institutional investors.

AHP as Bayesian Prior in Place of Black-Litterman

- Portfolios formed by the AHP method naturally have the properties of robustness, avoiding the major criticism of mean-variance portfolios.
  - AHP portfolios are also not subject to the criticisms leveled by Michaud et. al. at the Black-Litterman method. In AHP, the solution is closely related to the first principal component of the combined AHP matrix. This has the effect of filtering out competing aspects of the problem to arrive at a sensible compromise solution.
  - In addition, the AHP portfolios themselves are formed in the context of a much richer set of objectives than just risk and return. Those objectives could encompass liquidity, income needs, taxation and fee structures. As such, AHP portfolios are highly efficient as Bayesian priors for the formation of mean-variance portfolios.
  - The robust nature of the AHP frontier is illustrated in Exhibit 6 of Bolster and Warrick (2008). The portfolios have high diversification and very smooth weight transition as we move across the spectrum of investor risk tolerance. This exhibit is very similar to Exhibit 6.2 of Michaud and Michaud (2008).

The AHP Frontier

Exhibit 6: Suitable Model Portfolios for Aggressiveness Variations



Identifying Risk Aversion

- One of the most vexing problems for wealth investors and their advisors is to answer the question "How aggressive should my portfolio be?" The traditional mean-variance utility function from Markowitz and Levy (1979) is stated as:

  $$U = \alpha - \lambda \sigma^2$$

  U is the investor's utility (satisfaction) from the portfolio

  α is the mean return

  λ is the investor's risk aversion

  σ is the standard deviation of return

  The λ value represents the slope of a line drawn tangent to the efficient frontier so as to intersect at the chosen portfolio.

Using AHP to Infer Risk Aversion

- The problematic aspect of the MV process is we need to know λ before we can select our optimal portfolio.
  - Even in this idealized case where we have reduced the complex objectives of high net-worth investors to just risk and return it is heroic to assume that investors can meaningfully articulate a numeric value for risk aversion.
  - The AHP method can again be used to add richness to the investor decision process. To the extent that AHP methods can identify a single specific portfolio as appropriate for a given household, we can use our normal capital market assumptions to estimate the value of the return and risk of that portfolio.
  - If we assume the AHP portfolio is "close enough" to the efficient frontier, we can then ask ourselves "If we are happy holding the portfolio derived from the AHP process, what is the value of λ that would have selected this point on the efficient frontier?"

Why Do We Need Explicit Risk Aversion?

- A key decision that an investor must make about asset allocation is how to change the allocation when market conditions change
  - If we have an explicit value for λ, we have a clear path of action. If we believe that expected returns had changed for certain asset classes (e.g. Asian equities, or European sovereign bonds) this would have the effect of changing the shape of the Markowitz efficient frontier.
  - However, there would be no reason that such a change in our return expectations would change the investor's risk aversion. We would then apply the same tangency slope to the revised efficient frontier to find our new mean-variance portfolio (hopefully using the AHP portfolio as a prior).
  - A change in risk aversion would be appropriate if the financial circumstances of a household might change due to particular events such as changes in age, marriage, divorce or inheritance.
  - By helping to explicitly identify appropriate levels of investor risk aversion, the process of revising the asset allocation is neatly divided *into situations that involve changes in the state of the world (i.e. capital market expectations) and changes in the circumstances of the investor.*

Discretionary Wealth Hypothesis

- In the prior section, the discussion focused the frequent inability of investors to explicitly state their risk aversion. One approach to this problem is the “DWH” developed in Wilcox (2003).
  - Stated simply, the DWH says “investors should set their risk aversion so as to maximize the expectation of their geometric mean return on the portion of their wealth that they can afford to lose (but no more)”.
  - The implication of this assertion is that investors should operate as to put a “floor” value on the wealth, and that the value of the floor is equal to their financial liabilities.
  - The financial condition of a household is expressed in a “life balance sheet” wherein the asset side of the balance sheet contains all available financial resources of the household including the present value of expected future savings, and the liability side of the balance sheet contains all liabilities inclusive of the present value of expected future consumption expenditure Like any balance sheet, the surplus of assets over liabilities in the net worth of the household.

Getting to Risk Aversion

- Wilcox shows that under very reasonable assumptions, the optimal value of the risk aversion parameter l is linearly is given by:

  $\lambda^* = T / (2 * (T-L))$

  λ^* = optimal value of risk aversion

  T = the total assets on the life balance sheet

  L = the total liabilities on the life balance sheet

  If we have a sufficient knowledge of the household circumstances, we not only know the best value for λ, but also how this *value should change over time as the financial condition of the household changes*, both of changes in the market value of assets and family events such as inheritance or divorce.

The Rich are "Different"

- While the DWH is a powerful conceptual tool for the management of asset allocation, there is often an impediment to use of this technique for very wealthy households.
  - For such families, the value of explicit liabilities will approach zero, and the value of the ratio of assets to net worth will approach one. Under such a circumstance, the DWH would suggest that the very wealthy should invest quite aggressively, since they can afford to absorb large losses without meaningful reduction in their ability to support their lifestyle.
  - In practice, most very wealthy investors prefer to hold relatively conservative portfolios, and the "preservation of capital" is the key phrase in the vocabulary of their advisors.
  - This behavior can be reconciled to the DWH in a simple fashion.   Very wealthy investors know that their wealth will last beyond their lifetime, and they make asset allocation choices in a fashion consistent with *an implicit liability to provide financially for future generations* of the family or favored charitable causes.

Reconciliation of AHP and DWH

- High net-worth investors typically have only a relatively vague notion of how their asset allocation policies may impact a long-run legacy.
  - One way to get clarity is to think of the financial resources of a wealthy household as an endowment that is intended to provide perpetual income.
  - If we employ AHP portfolios to estimate λ as described in the prior section, we also know the corresponding value of the ratio of assets to net worth per the DWH. We can then algebraically solve for the present value of the endowment perpetuity that is consistent with our asset allocation policy.
  - If only liability is the present value of the implicit endowment income stream, the present value of the liabilities is equal to

$$L = D / K$$

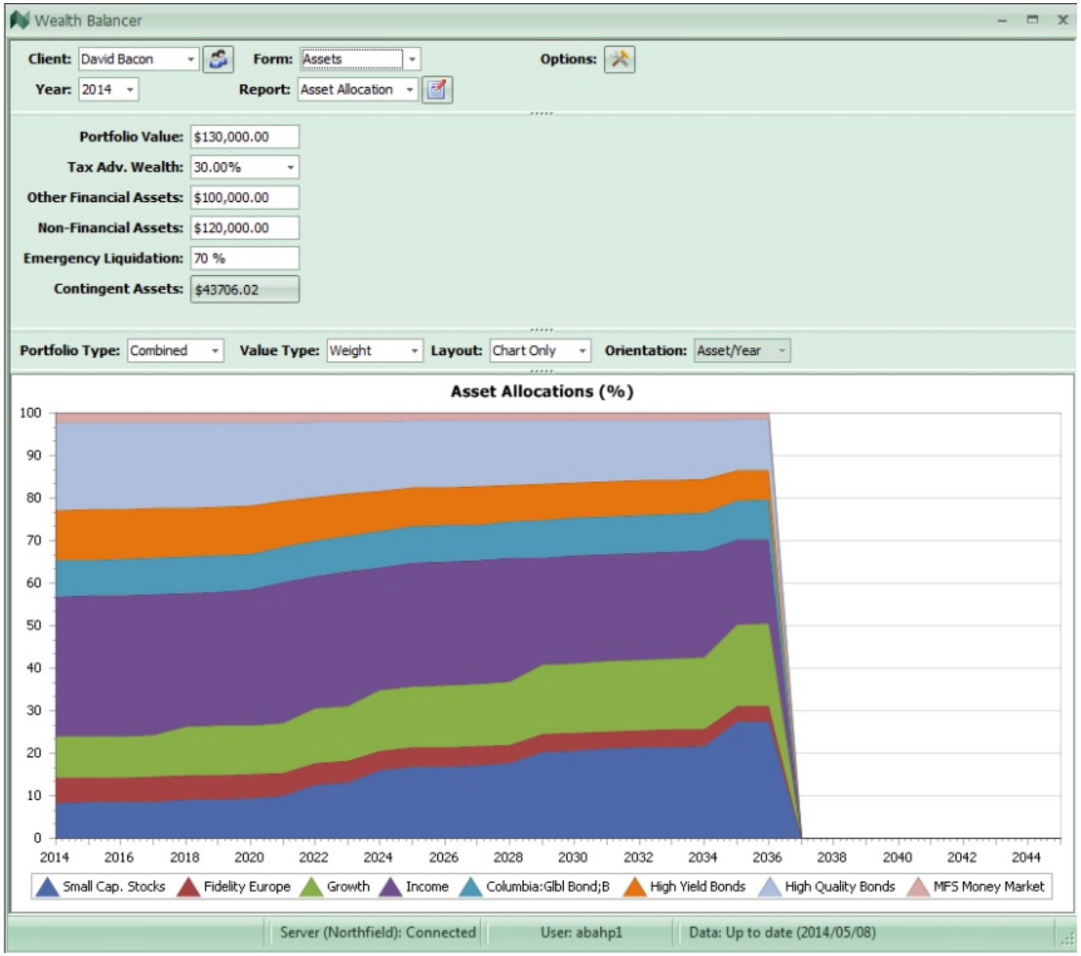
D is the per period income to be produced by the endowment

K is the decimal discount rate

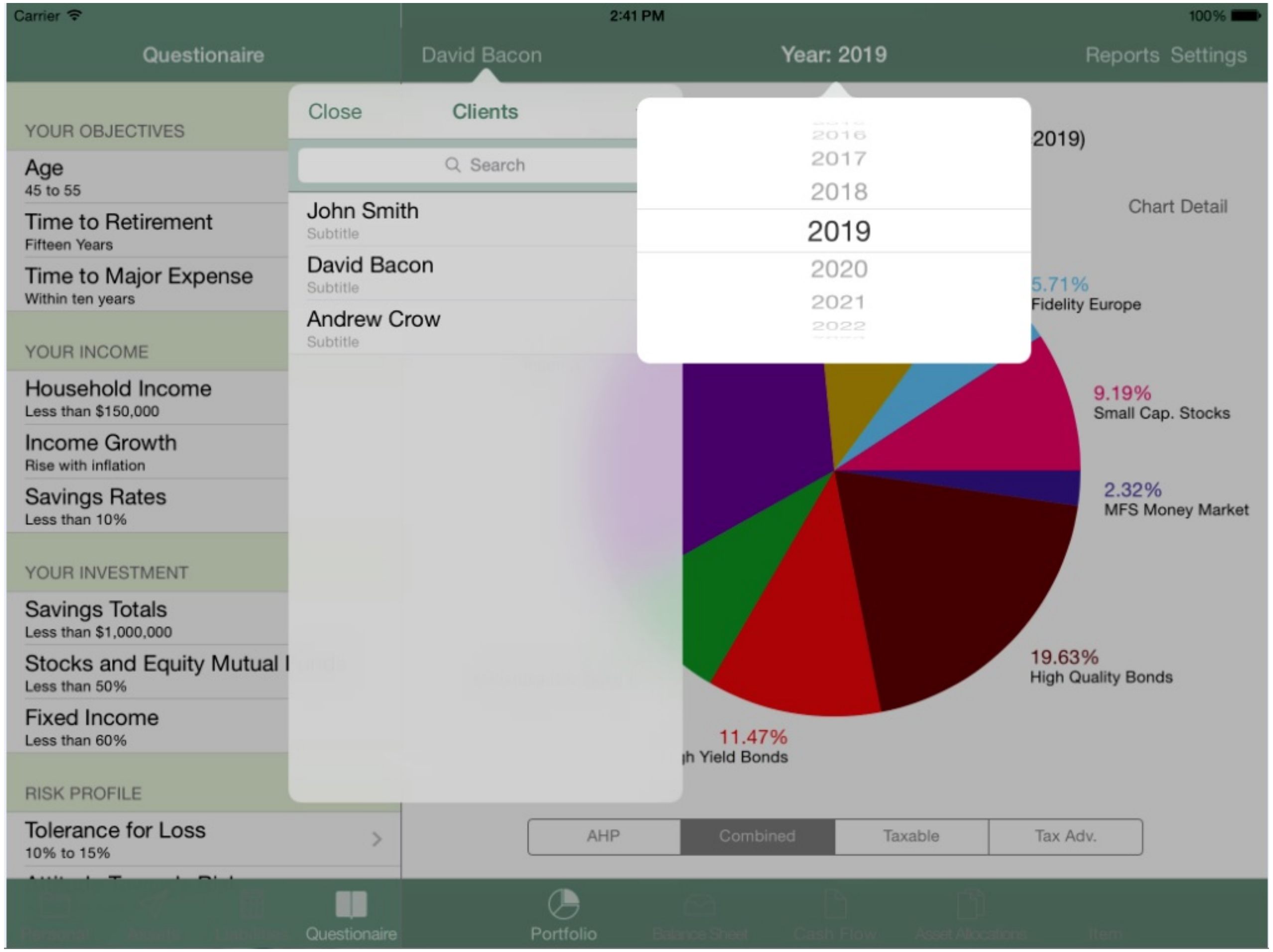
Conclusion

- The Analytic Hierarchy Process is a powerful non-parametric method for making optimal decisions in a wide array of problems including the asset allocation. It is particularly well suited to the complex objectives of high net-worth households.
- Asset allocation portfolios obtained from the AHP approach have a high degree of robustness which makes them an ideal "prior" in Bayesian techniques intended to address solution instability in mean-variance optimized portfolios.
- To the extent that AHP brings focus to the needs and wants of the investor, rather than on our expectations of capital markets, the technique can be usefully employed to help quantify investor risk aversion which is otherwise unobservable.
- Finally, we can combine our improved specification of investor risk aversion to bring clarity to the multigenerational endowment aspects of financial situation of high net-worth households.

Forward Curve of Allocations



IPad Version with Scrolling Forward Allocation



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