Understanding the Distribution of Hedge Fund Returns

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Key Points for this Talk

• The distribution of hedge fund returns to investors is structurally non-normal even without the use of derivatives.

• Distributions of hedge fund returns usually have negative skew and positive excess kurtosis (fat tails). These higher moments arise from several causes:
  – The “Central Paradox of Active Management”
  – The distribution of security returns over short intervals
  – Many hedge funds are structurally short volatility

• Fund of fund structures are not a guarantee against the existence of higher moments
The Central Paradox of Active Management

• All active fund managers must believe their future returns will be above benchmark (or peer group average) in order to rationally pursue active management, yet it is axiomatically true that roughly half of active managers must produce below average results.

• This means that for the hedge fund investor, there are two sources of risk.
  – The volatility of fund returns around the mean of the fund return
  – The possibility you hired a bad manager and the true mean of future fund returns is negative.
A Picture for Uncertainty of Mean

- The resulting distribution is bimodal with modes at $\alpha_p$ and $-\alpha_p$.

- Distribution has negative skew and positive excess kurtosis relative to the manager’s expectation of $\alpha_p$. 
Strategy Risk

• We will call the non-zero potential to invest with a bad manager “strategy risk”

• There are both multiplicative and additive approaches for adjusting the perceived risk of a fund so as to include strategy risk
  – For traditional active managers, the perceived risk often increases by 30% or more, resulting in decline in the expectation of the information ratio.
  – For hedge funds that claim very high IR (e.g. Bernie Madoff) the perceived increase in risk can be 300% or more. Bernie’s fund would not have been a great reward/risk tradeoff even if it wasn’t a fraud (which we happened to figure out in early 1999 in one day).
Security Return Distributions

• While traditional portfolio theory assumes that returns for equity securities and market are normally distributed, there is a vast amount of empirical evidence that the frequency of large magnitude events seems much greater than is predicted by the normal distribution, especially over shorter time horizons (i.e. days rather than months).

• Three broad schools of thought:
  – Equity returns have stable distributions of infinite variance.
  – Equity returns have specific, identifiable distributions that have significant kurtosis (fat tails) relative to the normal distribution (e.g. a gamma distribution).
  – Distributions of equity returns are normal at each instant of time, but look fat tailed due to time series fluctuations in the variance.
Sad Abuse of the Central Limit Theorem

• The “normal” or Gaussian distribution is widely observed in nature and has special properties in mathematics.

• The expectation of having a normal distribution arises from the CLT which says:
  – The sum of any set of very large (near infinite) number of independent distributions will be normal.
  – Lack of observable correlation is a necessary but not sufficient condition for the CLT to hold.

• It’s hard to see how this applies to financial markets where every trade has to have two sides and everyone is reacting to the release of the same information.
Investor Response to Information

• The relative response to “news” and “announcements”
  – Abraham and Taylor (1993)

• Jones, Lamont and Lumsdaine (1998) on US bonds
  – Total returns for long bonds and Treasury bills are not different if announcement days are removed from the data set

• Brown, Harlow and Tinic (1988) provide a framework for asymmetrical response to “good” and “bad” news
  – Good news increases projected cash flows, bad news decreases
  – All new information is a “surprise”, decreasing investor confidence and increasing discount rates
  – Upward price movements are muted, while downward movements are accentuated

• diBartolomeo, Mitra and Mitra (2009)
## How Fat Can Tails Be?

### Probability of a realization > n standard deviations away from mean

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<td>1/1.75 MIL</td>
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Short Volatility Strategies

• Many hedge fund strategies are based on “value” or “momentum” driven security selection strategies.
  – Momentum strategies buy on price strength and sell on weakness like CPPI (Black and Perold, 1992) applied to the cross-section of stock returns. CPPI mimics being long a put option on the underlying asset (plus a long position in the underlying). **Long option strategies have positive skew**
  – If momentum strategies are comparable to being long an option, then anti-momentum strategies (value?) must be comparable to being short an option, so low volatility conditions would be most favorable. **Like all short option strategies, value has negative skew**
  – Performance fees on hedge funds give managers incentive to use value strategies which create positive returns most of the time, but take rare large losses (negative skew).
Consider the correlation of two hedge funds. The two funds happen to have offices in the same building. The two PMs meet each morning at the lobby coffee shop. Each morning they flip a coin. If the coin comes up “heads” they hold exactly the same positions for that day. If the coin comes up “tails” they go long and short against each other.

On the days they hold the same positions, their return correlation will be one. On the days the two funds go long and short against each other, their correlation will be negative one. On average, their correlation is zero, giving the false impression that they act independently.
Convert the Four Moment Problem to Just Two

• Cornish-Fisher (1937)
  – We can convert any four moment distribution into an approximately equivalent
two moment distribution
  – We adjust the mean and standard deviation to best fit the desired percentile (the
  \( P \) from the VaR/CVaR definitions) for the subject distribution

• We can also estimated an adjusted volatility \( V^* \)

\[
V^* \sim (S^2 - (200/RAP) [(2/3)* MS^3 + (1/2)* KS^4])^{0.5}
\]

In the absence of explicitly stated risk aversion we approximate \( RAP = 6 * S \)
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