The Black-Litterman Model: Active Risk Targeting and the Parameter Tau

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Overview

- In the Black-Litterman model, the parameter tau (τ) determines the overall weight given to active versus passive investment views
  - Tau originates from the seminal Bayesian derivation of the model; despite its importance, tau has proved to be a very confusing aspect of Black-Litterman in terms of interpretation, estimation, and implementation
  - We offer a simple interpretation of tau, show that it is directly related to the level of active risk implicit in Black-Litterman expected returns, and is easily calibrated so that Black-Litterman expected returns produce a portfolio with a targeted level of active risk
- We introduce an alternative derivation of Black-Litterman that affords a more direct way of targeting active risk that doesn't require a specific value for tau
  - The derivation clearly shows that portfolio construction using the Black-Litterman model is equivalent to creating a mean-variance optimal portfolio of active strategies that is then overlaid onto a benchmark portfolio
  - By targeting active risk directly, users of the Black-Litterman model can effectively ignore tau

Introduction

- Introduced 25 years ago, The Black-Litterman (BL) model was developed to mitigate problems related to mean-variance optimization (MVO)
  - Unconstrained MVO portfolios are typically (1) very sensitive to expected return inputs, (2) concentrated in just a few assets, and (3) not reflective of underlying active investment views
- BL incorporates passive "equilibrium" expected returns and active "views"
  - Equilibrium expected returns proxy for consensus expected returns associated with a benchmark
  - Views are portfolio-level expected returns associated with active investment strategies
- BL converts views into asset-level active expected returns that are blended with equilibrium expected returns
  - Unconstrained MVO holdings for assets not included in any active views will be equal to benchmark weights
  - Active weights will differ from the benchmark in proportion to the view portfolios

Black-Litterman is apparently well-known and widely used...

- Google Scholar search yields 5,370 results for BL; 1,374 citations for "Global Portfolio Optimization" (Black and Litterman, 1992)
- Asset managers: Barclays Wealth and Investment Management, Goldman-Sachs, and UBS
- Investment advisors: Standard and Poor's Investment Advisory Services (SPIAS) and Betterment
- Investment software providers: offered as a portfolio construction tool from Morningstar Direct and Zephyr Associates
- Sell-side: promoted in research from Deutsche Bank Securities and J.P. Morgan
- "...many trillions of institutional and hedge fund dollars are invested in Black-Litterman optimized portfolios." (Michaud, 2012)

...but is it well understood?

- BL literature includes publications such as:
  - “The Intuition Behind Black-Litterman Model Portfolios” (He and Litterman, 1999)
  - “A Demystification of the Black-Litterman Model” (Satchell and Scowcroft, 2000)
  - “A Step-by-Step Guide to the Black-Litterman Model” (Izadorek, 2004)
  - “The Black-Litterman Model Explained” (Cheung, 2010)
  - “Deconstructing Black-Litterman” (Michaud, Esch, and Michaud, 2013)
  - “Reconstructing the Black-Litterman Model” (Walters, 2014)
- BL is typically analyzed in the context of Bayesian statistical methods
  - Emphasis on its Bayesian statistical underpinnings likely obfuscates the practical workings of BL for many prospective (and even current) users of the model

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Black-Litterman expected returns

- The BL "master equation":

$$r_{BL} = \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q\right]$$

r_{BL} is the N x 1 vector of expected returns;

τ is a parameter that reflects the level of confidence in the equilibrium expected returns;

Σ is the N x N matrix of asset return covariances;

P is the K x N view matrix of asset portfolio weights for K active investment strategies;

Ω is the K x K diagonal matrix containing measures of confidence in each strategy;

Π is the N x 1 vector of equilibrium expected returns; and

Q is the K x 1 vector of views, which are the expected returns for the active strategies.

Deriving Black-Litterman (two approaches out of many)

- Theil's mixed estimator (generalized least squares derivation):

$$y = xr_{BL} + e,\ e \sim N(0, V)$$

$$y = \begin{pmatrix} \Pi \\ Q \end{pmatrix},\ x = \begin{pmatrix} I \\ P \end{pmatrix},\ e = \begin{pmatrix} u \\ v \end{pmatrix},\ V = \begin{pmatrix} \tau\Sigma & 0 \\ 0 & \Omega \end{pmatrix}$$

$$r_{BL} = \left(x'V^{-1}x\right)^{-1} x'V^{-1}y$$

- Simultaneous least squares derivation:

$$\min_{r_{BL}} \frac{1}{2}\left[\left(r_{BL} - \Pi\right)'\left(\tau\Sigma\right)^{-1}\left(r_{BL} - \Pi\right) + \left(Q - Pr_{BL}\right)'\Omega^{-1}\left(Q - Pr_{BL}\right)\right]$$

- Both are optimization problems, but do they convey any real investment intuition...?

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Alternative derivation of Black-Litterman

- The BL master equation can also be written

$$\begin{aligned} r_{BL} &= \Pi + \left[\tau\Sigma P'\left(\tau P\Sigma P' + \Omega\right)^{-1}\right]\left(Q - P\Pi\right) \\ &= \Pi + \Lambda\alpha_S \\ &= \Pi + \alpha_{BL} \end{aligned}$$

BL expected returns are equal to the sum of passive and active expected returns

- Unconstrained MVO: $\max_{h_P} \ h_P' r_{BL} - \frac{\delta}{2} h_P' \Sigma h_P$

$$\begin{aligned} \rightarrow \ h_P &= \delta^{-1}\Sigma^{-1} r_{BL} \\ &= \delta^{-1}\Sigma^{-1}\Pi \ + \ \delta^{-1}\Sigma^{-1}\alpha_{BL} \\ &= h_B + h_A \end{aligned}$$

BL portfolio holdings are equal to benchmark weights plus a vector of active weights

- Note: δ is a risk aversion parameter

Equilibrium expected returns

- Equilibrium expected returns, also referred to as implied returns, are calculated using *reverse optimization*:

$$h_B = \delta^{-1}\Sigma^{-1}\Pi \quad \Rightarrow \quad \Pi = \delta\,\Sigma\,h_B$$

Equilibrium expected returns are engineered to produce benchmark weights when used in unconstrained MVO

- In Bayesian terminology, equilibrium returns are the “prior” estimates of expected returns
- Equilibrium returns act as an anchor for active investment views and obviate the need to calculate expected returns for all assets individually
- Original BL derivation uses market capitalization weights that correspond to the equilibrium assumptions of the CAPM; in practice, any benchmark can be used

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Active expected returns

- O'Toole (2013) uses reverse optimization to derive BL alphas:
  - Strategy covariance matrix: $\Sigma_S = P\Sigma P'$
  - Confidence-adjusted strategy alphas: $\tilde{\alpha}_S = \left[\tau\Sigma_S\left(\tau\Sigma_S + \Omega\right)^{-1}\right]\alpha_S$
- Unconstrained MVO: $\max_w \; w'\tilde{\alpha}_S - \frac{\delta}{2} w'\Sigma_S w \;\rightarrow\; w = \delta^{-1}\Sigma_S^{-1}\tilde{\alpha}_S$
- Calculate asset-level active holdings by multiplying each view portfolio by its corresponding MVO weight and sum the scaled holding across strategies:

$$h_A = P'w = \begin{pmatrix} p_{11} \\ \vdots \\ p_{N1} \end{pmatrix} w_1 + \begin{pmatrix} p_{12} \\ \vdots \\ p_{N2} \end{pmatrix} w_2 + \cdots + \begin{pmatrix} p_{1K} \\ \vdots \\ p_{NK} \end{pmatrix} w_K$$

- Apply reverse optimization to the active holdings:

$$\alpha_{BL} = \delta\Sigma h_A = \left[\tau\Sigma P'\left(\tau P\Sigma P' + \Omega\right)^{-1}\right]\left(Q - P\Pi\right)$$

BL alphas are designed to produce the same unconstrained MVO active holdings as those generated from MVO at the strategy level

Practitioner interpretation of Black-Litterman

- Portfolio construction using the Black-Litterman model is equivalent to creating a mean-variance optimal portfolio of active strategies that is then overlaid onto a benchmark portfolio
  - Unconstrained MVO holdings for assets not included in any active views will be equal to benchmark weights
  - Active weights will differ from the benchmark in proportion to the view portfolios
- Mean-variance mechanics are embedded in Black-Litterman expected returns
  - The active portion of Black-Litterman expected returns is associated with a specific MVO active portfolio
- Consistent use of the risk model (covariance matrix) promotes stable results vis-à-vis traditional mean-variance optimization

Example: He and Litterman two market views case

	Benchmark Weights, h_B (%)	Annualized Volatilities (%)	Correlation Matrix: Australia	Canada	France	Germany	Japan	UK	USA	Equilibrium Expected Returns, $\Pi = \delta\Sigma h_B$ (%)
Australia	1.6	16.0	1							3.94
Canada	2.2	20.3	0.488	1						6.92
France	5.2	24.8	0.478	0.664	1					8.36
Germany	5.5	27.1	0.515	0.655	0.861	1				9.03
Japan	11.6	21.0	0.439	0.310	0.355	0.354	1			4.30
UK	12.4	20.0	0.512	0.608	0.783	0.777	0.405	1		6.77
USA	61.5	18.7	0.491	0.779	0.668	0.653	0.306	0.652	1	7.56

	View Matrix*, P' (%): Europe/UK	North America
Australia	0	0
Canada	0	100.0
France	-29.5	0
Germany	100.0	0
Japan	0	0
UK	-70.5	0
USA	0	-100.0

*Note that this is the transpose of the view matrix.

Strategy Covariance Matrix, $\Sigma_S = P\Sigma P'$ (%)	Europe/UK	North America
Europe/UK	213.0	20.1
North America	20.1	170.3

Strategy Confidence, Ω (%)	Europe/UK	North America
Europe/UK	10.6	0
North America	0	8.5

	Strategy Views, Q (%)	Equilibrium Views, $P\Pi$ (%)
Europe/UK	5.00	1.79
North America	3.00	-0.64

	Strategy Alphas, $\alpha_S = Q - P\Pi$ (%)	Adjusted Alphas, $\tilde{\alpha}_S = \tau\Sigma_S(\tau\Sigma_S+\Omega)^{-1}\alpha_S$ (%)
Europe/UK	3.21	1.71
North America	3.64	1.89

	Strategy MVO Weights, $w = \delta^{-1}\Sigma_S^{-1}\tilde{\alpha}_S$ (%)
Europe/UK	28.2
North America	41.1

Risk Aversion Parameter, $\delta = 2.5$

Strategy Correlation, $\rho = 10.5\%$

Ann. Active Risk, $\sigma_A = 7.1\%$

Ann. Portfolio Risk, $\sigma_P = 18.8\%$

Tau, $\tau = 0.05$

	Active Holdings, $h_A = P'w$ (%)	Portfolio Holdings, $h_P = h_B + h_A$ (%)	BL Expected Returns, $r_{BL} = \delta\Sigma h_P$ (%)	BL Alphas, $\alpha_{BL} = \delta\Sigma h_A = r_{BL} - \Pi$ (%)
Australia	0	1.6	4.42	0.48
Canada	41.1	43.3	8.73	1.81
France	-8.3	-3.1	9.48	1.12
Germany	28.2	33.7	11.21	2.18
Japan	0	11.6	4.62	0.31
UK	-19.9	-7.5	6.97	0.20
USA	-41.1	20.4	7.48	-0.08

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The parameter tau (τ)

- τ has proved to be a particularly confusing aspect of the BL model
  - τ is a non-negative scalar that reflects an overall level of confidence in the active views versus the equilibrium expected returns
  - In Bayesian terms, τ measures the *subjective* degree of uncertainty as to how precisely the equilibrium returns (the expected return priors) have been estimated
  - τ is a critical component of BL as it effectively determines the overall weight placed on the active views relative to the equilibrium expected returns
- The literature is replete with conflicting guidance as to how to interpret and quantify τ, and includes some harsh criticism of BL related to τ:
  - “The adjustment of τ to attain investability is an intervention which contaminates the rigor of the analysis and must be viewed as an *ad hoc* correction of a flawed procedure, and a major departure from rigorous statistical analysis.” (Michaud, Esch, and Michaud, 2013)
  - “...Black-Litterman is a failed scientific theory with vacuous investment value that should be ignored by the investment community as any serious solution to the instability and ambiguity of Markowitz optimization in practice.” (Michaud, 2012)

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Practitioner interpretation of τ

- A practical interpretation of τ is that it reflects an investor's degree of belief in efficient markets
  - Smaller values indicate less subjective uncertainty regarding market efficiency and shrink the weight on the active views toward zero
  - In the limiting case of $\tau = 0$ the weight on all of the active views is zero, and the investor will passively hold the "market" as represented by the benchmark portfolio
  - Larger values indicate belief in exploitable market inefficiencies, with higher values corresponding to greater confidence in the active views and more willingness to take on active risk
- Active managers typically have explicit targets for active risk based on the opportunity set, investment objectives, policy mandates, etc.

  τ provides a natural mechanism for controlling active risk, and having a specific target for active risk provides a sensible context for determining a value for τ

τ and active risk

- BL portfolio holdings can be separated into benchmark and active weights:

$$h_{BL} = h_B + h_A = \delta^{-1}\Sigma^{-1}\Pi + \delta^{-1}\tau P'\left(\tau P\Sigma P' + \Omega\right)^{-1}\left(Q - P\Pi\right)$$

Note that τ only enters the active component

- Active risk is equal to

$$\sigma_A = \sqrt{h_A'\Sigma h_A}$$
$$= \frac{\tau}{\delta}\sqrt{\left(Q - P\Pi\right)'\left(\tau P\Sigma P' + \Omega\right)^{-1} P\Sigma P'\left(\tau P\Sigma P' + \Omega\right)^{-1}\left(Q - P\Pi\right)}$$

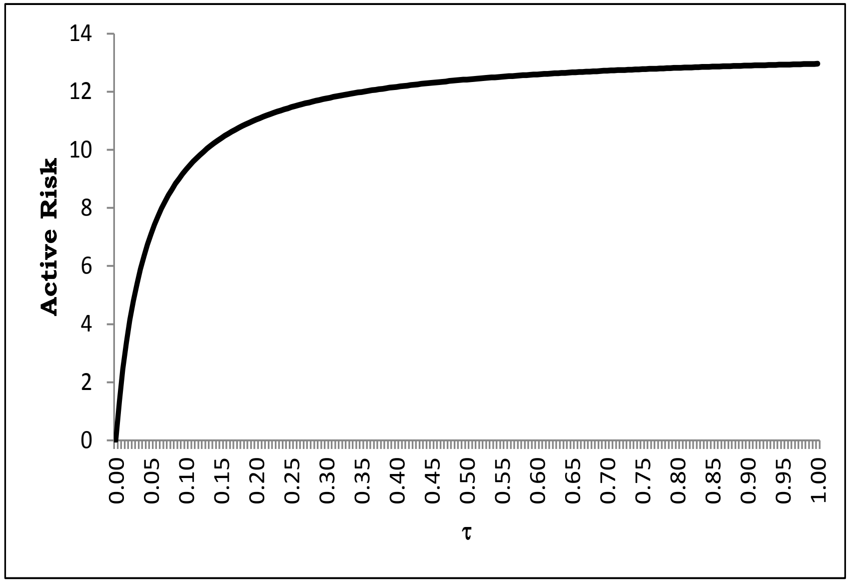
- Active risk is an increasing non-linear function of τ

Active risk increases at a decreasing rate with respect to τ, asymptotically approaching a well-defined upper bound:

$$\sigma_A^{MAX} = \lim_{\tau\to\infty}\sigma_A = \left(\frac{1}{\delta}\right)\sqrt{\left(Q - P\Pi\right)'\left(P\Sigma P'\right)^{-1}\left(Q - P\Pi\right)}$$

τ and active risk: two market views case

Active Risk as a Function of τ



Active Risk and τ

	Active Portfolio Holdings (%)			
	τ = 0.00786	τ = 0.05	τ = 0.353	τ → ∞
Australia	0	0	0	0
Canada	11.5	41.1	70.1	79.4
France	-2.4	-8.3	-13.9	-15.6
Germany	8.0	28.2	47.0	52.8
Japan	0	0	0	0
UK	-5.7	-19.9	-33.1	-37.2
USA	-11.5	-41.1	-70.1	-79.4
Ann. Risk (%)	2.0	7.1	12.0	13.5

- He and Litterman set τ = 0.05, which produces σ_A = 7.1%
- For an investor with 100% confidence in the active views, σ_A^{MAX} = 13.5%

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Targeting active risk directly

- There is a more direct way to target active risk that doesn't require a specific value for τ and for which there is no upper bound on active risk
- Recall that the active strategy MVO weights were solved for using δ, the same risk aversion parameter used to derive the equilibrium expected returns associated with the benchmark:

$$\max_w \; w'\tilde{\alpha}_S - \frac{\delta}{2} w'\Sigma_S w \;\longrightarrow\; w = \delta^{-1}\Sigma_S^{-1}\tilde{\alpha}_S$$

- O'Toole (2013, 2017) incorporates an active risk aversion parameter, ϕ, that corresponds to a targeted level of active risk, $\sigma_{A,T}$:

$$\max_{\hat{w}} \; \hat{w}'\tilde{\alpha}_S - \frac{\phi}{2}\left(\sigma_{A,T}^2 - \hat{w}'\Sigma_S\hat{w}\right) \;\longrightarrow\; \hat{w} = \phi^{-1}\Sigma_S^{-1}\tilde{\alpha}_S \, ,$$

$$\phi = \frac{\sqrt{\tilde{\alpha}_S'\Sigma_S^{-1}\tilde{\alpha}_S}}{\sigma_{A,T}} = \left(\frac{\tau}{\sigma_{A,T}}\right)\sqrt{(Q-P\Pi)'\left(\tau P\Sigma P'+\Omega\right)^{-1}P\Sigma P'\left(\tau P\Sigma P'+\Omega\right)^{-1}(Q-P\Pi)}$$

The active risk aversion parameter, ϕ, will adjust for different values of τ

Risk-targeted Black-Litterman alphas

- Asset-level active holdings using risk-targeted MVO weights:

$$\hat{h}_A = P'\hat{w} = \phi^{-1}\tau P'\left(\tau P\Sigma P' + \Omega\right)^{-1}\left(Q - P\Pi\right)$$

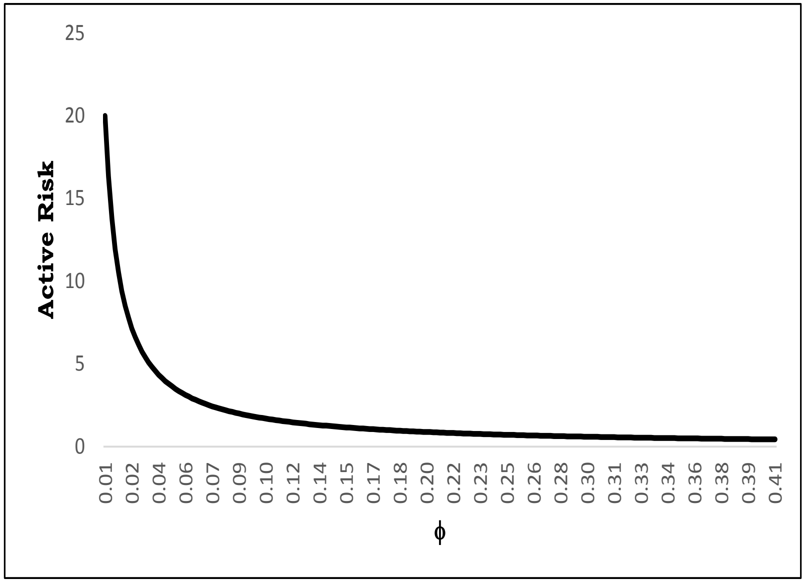
- Use reverse optimization to calculate risk-targeted BL alphas:

$$\hat{\alpha}_{BL} = \delta\Sigma\hat{h}_A = \left(\frac{\delta}{\phi}\right)\left[\tau\Sigma P'\left(\tau P\Sigma P' + \Omega\right)^{-1}\right]\left(Q - P\Pi\right) = \left(\frac{\delta}{\phi}\right)\alpha_{BL}$$

The active component of BL expected returns can simply be scaled by the ratio of benchmark risk aversion to active risk aversion in order to target a particular level of active risk

Targeting active risk directly: two market views case

Active Risk as a Function of ϕ (τ = 0.05)



Active Risk and ϕ

	Active Portfolio Holdings (%)				
τ = 0.05	ϕ = 0.0888	ϕ = 0.0250	ϕ = 0.0148	ϕ = 0.0131	ϕ = 0.0104
τ = 1	ϕ = 0.1620	ϕ = 0.0456	ϕ = 0.0270	ϕ = 0.0240	ϕ = 0.0191
τ = 5	ϕ = 0.1678	ϕ = 0.0473	ϕ = 0.0280	ϕ = 0.0249	ϕ = 0.0197
τ = 20	ϕ = 0.1690	ϕ = 0.0476	ϕ = 0.0282	ϕ = 0.0250	ϕ = 0.0199
Australia	0	0	0	0	0
Canada	11.5	41.1	70.1	79.4	98.5
France	-2.4	-8.3	-13.9	-15.6	-20.0
Germany	8.0	28.2	47.0	52.8	67.5
Japan	0	0	0	0	0
UK	-5.7	-19.9	-33.1	-37.2	-47.6
USA	-11.5	-41.1	-70.1	-79.4	-98.5
Ann. Risk (%)	2.0	7.1	12.0	13.5	17.0

- Higher levels of active risk correspond to lower levels of active risk aversion
- τ is subsumed by the active risk aversion parameter: ϕ adjusts for τ to generate portfolios with specific active risk targets, and there is no upper bound on active risk

Targeting active risk directly eliminates the need to choose a specific value for τ

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Conclusion

- There is apparent persistent confusion over certain aspects of Black-Litterman expected returns, with a number of publications offering various explanations and clarifications as to how the model works in practice
- The parameter tau has proved to be especially perplexing and contentious, with many authors offering widely varying suggestions as to how this important component of the model should be interpreted and quantified
- We show there is a direct relationship between tau and active risk, and tau can be calibrated to produce mean-variance optimal Black-Litterman portfolios with targeted levels of active risk, but only up to a maximum amount
- Our alternative derivation of Black-Litterman clearly reveals the mean-variance mechanics of the model while affording a more direct way to target active risk, with no need to set a specific value for tau and no upper bound on active risk

Investors who target active risk directly using the Black-Litterman model need not concern themselves with tau at all

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