Active Risk and Information Ratio of Active Management

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Introduction

Quantitative Equity Research
Previous Research

- Grinold, R.C. 1994, “Alpha Is Volatility Times IC Times Score.” *JPM*
- Sorensen, E.H., E. Qian, R. Hua, and R. Schoen. 2004. “Multiple Alpha Sources and Active Management.” *JPM*
- Qian & Hua. 2004. “Active Risk and Information Ratio of Active Management.” *submitted*
Key Practical Questions

- How to select quantitative factors
  - Information coefficient (IC)
- How to translate factor score into alpha input to an optimizer
  - “Alpha is volatility times IC times score”
- How to combine multiple alpha factors
  - IC & factor correlation
- What is the information ratio (IR) of a quantitative equity strategy
- What is the impact of various practical constraints
Key Previous Results

- Fundamental Law of Active Management (FLOAM)
  \[ IR = IC_t \sqrt{N} \]

- Modified FLOAM with transfer coefficient
  \[ IR = TC \cdot IC_t \sqrt{N} \]

- Alpha is volatility times IC times score
  \[ \alpha_i = IC_t \sigma_i z_i \]
Active Risk

Should We Trust Risk Models?

5/5/2004
A “Missing” Question

- What is the active risk of quantitative equity strategy
  - The obvious answer: the tracking error target given by a risk model
- If that is “wrong”, then the problem might be with the risk model (Hartmann et al 2002)
  i. Estimation error in covariances in a risk model
  ii. Time varying nature of covariances
  iii. Serial auto-correlations of excess returns
  iv. Drift of portfolio weights over a given period
- What if the risk model is “perfect”?
- Why ex post tracking errors seem to ALWAYS exceed ex ante tracking error by a risk model
The Tale of Two Strategies

- The “perfect” strategy
  - It produces a stream of constant excess return every period, 1%, 1%, …, 1%, …
  - By definition, it has no active risk against benchmark
  - But a risk model would always tell us the strategy has active risk

- The bad strategy
  - Its excess returns are volatile, 1%, -1%, 1%, …, 
  - Its active risk could easily exceed the tracking error estimate by a risk model
Introducing Strategy Risk

- In addition to the generic risk model risk, each strategy has its own strategy risk.
- The strategy risk reflects the variability of the model’s ability to generate excess return.
- It can be measured by the standard deviation of IC.
- The true active risk of a strategy is a combination of model risk and its own strategy risk.
- The information ratio should take it into account.
  - This should not be too surprising since we already acknowledge the difference in average IC.
New Results on Active Risk

- True active risk consists of \( \sigma = \text{std}(IC)\sqrt{N}\sigma_{\text{model}} \)
  - Risk-model target tracking error
  - Strategy risk
  - Square root of the breadth

- The strategy risk is different for different strategies
  - Different factors
  - Different combination of factors

- In most cases, true active risk is greater than the risk-model tracking error
  - They are the same only if \( \text{std}(IC) = \frac{1}{\sqrt{N}} \)
  - This is true if strategy risk is purely sampling error
New Results on IR

- Average excess return remains roughly the same
  \[ \bar{\alpha}_t = \bar{IC}_t \sqrt{N} \sigma_{\text{model}} \]

- The information ratio is then average IC divided by standard deviation of IC
  \[ IR = \frac{IC}{\text{std}(IC)} \]

- The results applies to individual factors, different combinations of factors

- They are different for different stock markets, style groups, sectors
Quantitative Equity Models

A Portfolio Approach
A New Approach

- New approach to build quantitative equity models
- The new approach is descriptive in nature
- We derive the new results based on the approach
- It differs from the “normative” approach

FLOAM
A Normative Approach

Risk Model

Forecasts

Alpha factors

APT

MV Optimization

Optimal Weights

Actual Returns

Excess Return

APT

Alpha factors
Normative Versus Descriptive

- Derivation of fundamental law depends on normative framework of APT
  - Fundamental risk models are often based on APT framework
  - Expected excess returns of individual securities assumed to follow APT: “beta” times alpha factor

- Descriptive approach
  - Practitioners need risk model to do mean-variance optimization
  - But it is not necessary to APT will be true for each security
  - Based on “real” long-short portfolios
The New Approach

- Risk Model
- Alpha factors
- MV Optimization
- APT
- Optimal Weights
- Actual Returns
- Risk Factor Neutral
- Excess Return
Details

- Select a risk model – BARRA, Northfield, APT ...
- Select a alpha factor – valuation, momentum, profitability, ...
- Select a fixed trading period – monthly, quarterly, ...
- MV optimization chooses optimal long-short portfolio
  - With fixed target tracking error
  - With all risk factor neutralized
  - Exact solution exists for the optimization
- Compute single period alpha
- Compute multi-period statistics – average, standard deviation, IR, ...
Limitations and Extensions

- Focus on strategies exploiting only the stock-specific returns with no factor bets
  - Mathematical convenience
    - Exact solution for optimal weights
    - No need of historical factor return covariance matrices
  - Most quantitative strategies have minimal risks
- Analysis can be extended to strategies with factor bets
  - We can expect the same qualitative conclusion such as different strategy risk
  - The magnitudes of bias are not yet known
Single-period Results

- Optimal long-short portfolio weights
  \[ w_i = \lambda^{-1} \sum_{i=1}^{N} R_{i,t} F_{i,t} \]

- Single-period excess return
  \[ \alpha_t = \lambda_t^{-1} \sum_{i=1}^{N} R_{i,t} F_{i,t} \]

- Risk-adjust forecasts and risk-adjusted returns
  \[ F_i = \frac{f_i - l_1 - l_2 \beta_{i1} - \cdots - l_{M+1} \beta_{Mi}}{\sigma_i} \]
  \[ R_i = \frac{r_i - k_1 - k_2 \beta_{i1} - \cdots - k_{M+1} \beta_{Mi}}{\sigma_i} \]

- Single-period excess return in terms of IC and dispersions
  \[ \alpha_t = \lambda_t^{-1} (N-1) IC_t \text{dis} (R_t) \text{dis} (F_t) \]

- Final results
  \[ \alpha_t \approx IC_t \sqrt{N} \sigma_{\text{model}} \text{dis} (R_t) \]
Implications

- The analysis is based on realistic long-short portfolios.
- IC is correlation coefficient between risk-adjusted forecasts and risk-adjust return.
  - Not raw forecasts and raw returns.
  - Not regressed forecasts.
- We do not impose any relationship (linear) between individual returns and individual forecasts, i.e., no APT.
- For a consistent risk model, cross sectional dispersion of risk-adjusted return should be close to unity.
- Therefore \( \alpha_t \approx IC_t \sqrt{N} \sigma_{\text{model}} \).
Almost all alpha factors produce tracking error higher than the target of 5%. Some are significantly higher.
Persistence of Strategy Risk

Second half active risk without adjustment

Second half active risk with adjustment for strategy risk determined from the first half
Persistency of Strategy Risk

Strong positive correlation between two samples. Factors having higher (lower) strategy risk in the first half tends to higher (lower) strategy risk in the second half too.
Significant Difference

- Two valuation factors
  - Gross profit to enterprise value (GP2EV)
  - Forward earnings yield based on IBES FY1 consensus forecast (E2P)

- F-test shows that the variances of IC are significantly different at 5% level

<table>
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<th>Average Alpha</th>
<th>STD of Alpha</th>
<th>IR of Alpha</th>
<th>Average IC</th>
<th>STD of IC</th>
<th>IR of IC</th>
<th>Average dis(R)</th>
<th>Average N</th>
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<td>GP2EV</td>
<td>6.2%</td>
<td>6.9%</td>
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<td>E2P</td>
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A Factor Example

A decent factor
But the active risk is consistently above target
The risk-adjusted IC is much more stable, indicating a better information ratio.

The raw IC carries risk-factor bias.
Multi-factor Models

“Optimize” the IC
The Portfolio Approach

- Multiple forecasts for each N stocks
  - Previous research is unclear
  - Many practitioners resort to ad hoc weighting schemes based on intuition or judgment
- Our approach provides a quantitative framework for combining multiple factors
  - For a multi-factor model, it is still true that $IR = \frac{\overline{IC}}{\text{std}(IC)}$
  - We maximize the IR to find the optimal linear combination
  - The problem is very similar to optimal allocation among multiple active managers
- Use optimal weights with caution and common sense.
Important Inputs

- Each factor (manager)
  - Average IC, standard deviation of IC, IR
  - Analogous to expected alpha, active risk, IR
- Among factors (managers)
  - IC correlation: time series correlations between different ICs
  - Analogous to correlations between excess returns of different managers
  - The correlations between different factors are not of primary importance
  - Factor correlation is not the same as IC correlation
Two-factor Case

- Combined IC of a single period
  \[ IC_t = \frac{IC_{q,t} \cdot \omega_q + IC_{f,t} \cdot \omega_f}{\sqrt{1 - 2\omega_q \omega_f (1 - \rho_t)}} \]

- The factor correlation is only a scale parameter and when it is constant throughout time it has no effect on IR.

- Expected IR
  \[ IR \approx \frac{\overline{IC_t}}{std(\overline{IC_t})} = \frac{\overline{IC_{q,t}} \cdot \omega_q + \overline{IC_{f,t}} \cdot \omega_f}{\sqrt{\sigma_q^2 \omega_q^2 + 2\sigma_{f,q} \omega_f \omega_q + \sigma_f^2 \omega_f^2}} \]

- The optimal weights can be found

- For multi-factor case, there is a matrix solution
Summary

- Portfolio approach: simplified backtest
- Strategy risk contributes additionally to active risk. It can be measured by standard deviation of IC
- Use the proper definition of IC - the correlation between risk-adjusted forecasts and risk-adjusted returns, not the raw IC
- IR is often lower than what FLOAM predicts, even if there is no portfolio constraint and no model errors
- Select alpha factors based on average IC as well as standard deviation of IC
- Combine alpha factors based on the above as well as IC correlations