Active Risk and Information Ratio of Active Management

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Introduction

Quantitative Equity R[illegible]ch

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Previous Research

- Grinold, R.C. 1989, "Fundamental Law of Active Management." *JPM*
- Grinold, R.C. 1994, "Alpha Is Volatility Times IC Times Score." *JPM*
- Grinold & Kuhn, 1999, *Active Portfolio Managemen*[illegible] McGraw-Hill, New York
- Grinold, R.C., and R.N. Kahn. 2000. "The Efficiency Gai[illegible]ng-Short Investing." *FAJ*
- Clarke, R., H. de Silva, and S. Thorley. 2002. "Portfolio Const[illegible] the Fundamental Law of Active Management." *FAJ*
- Sorensen, E.H., E. Qian, R. Hua, and R. Schoen. 2004. "Multiple [illegible] Sources and Active Management." *JPM*
- Qian & Hua. 2004. "Active Risk and Information Ratio of Active Management." *submitted*

Key Practical Questions

- How to select quantitative factors
  - Information coefficient (IC)
- How to translate factor score into alpha input to an optimizer
  - "Alpha is volatility times IC times score"
- How to combine multiple alpha factors
  - IC & factor correlation
- What is the information ratio (IR) of a quantitative equity strategy
- What is the impact of various practical constraints

Key Previous Results

- Fundamental Law of Active Management (FLOAM)

$$IR = \overline{IC_t}\sqrt{N}$$

- Modified FLOAM with transfer coefficie

$$IR = TC \cdot \overline{IC_t}\sqrt{N}$$

- Alpha is volatility times IC times score

$$a_i = IC_t s_i z_i$$

Active Risk

Should We Trust Risk [illegible]dels?

A “Missing” Question

- What is the active risk of quantitative equity strategy
  - – The obvious answer: the tracking error target given by a risk model
- If that is “wrong”, then the problem mig[illegible] be with the risk model (Hartmann et al 2002)
  - i. Estimation error in covariances in a risk model
  - ii. Time varying nature of covariances
  - iii. Serial auto-correlations of excess returns
  - iv. Drift of portfolio weights over a given period
- What if the risk model is “perfect”?
- Why ex post tracking errors seem to ALWAYS exc[illegible] ex ante tracking error by a risk model

The Tale of Two Strategies

- The “perfect” strategy
  - It produces a stream of constant excess return every period, 1%, 1%, …, 1%, …
  - By definition, it has no active risk against bench
  - But a risk model would always tell us the strategy i risk
- The bad strategy
  - Its excess returns are volatile, 1%, -1%, 1%, …,
  - Its active risk could easily exceed the tracking error estima a risk model

Introducing Strategy Risk

- In addition to the generic risk model risk, each strategy has its own strategy risk
- The strategy risk reflects the variability o[illegible]odel's ability to generate excess return
- It can be measured by the standard deviation o[illegible]
- The true active risk of a strategy is a combinatio[illegible] model risk and its own strategy risk
- The information ratio should take it into account
  - This should not be too surprising since we already acknowledge the difference in average IC

New Results on Active Risk

- True active risk consists of $s = \text{std}(IC)\sqrt{N}s_{\text{model}}$
  - Risk-model target tracking error
  - Strategy risk
  - Square root of the breadth
- The strategy risk is different for different strat
  - Different factors
  - Different combination of factors
- In most cases, true active risk is greater than the ris
  model tracking error
  - They are the same only if $\text{std}(IC) = 1/\sqrt{N}$
  - This is true if strategy risk is purely sampling error

New Results on IR

- Average excess return remains roughly the same

$$\overline{a_t} = \overline{IC_t}\sqrt{N}\,s_{\text{model}}$$

- The information ratio is then average IC divided by standard deviation of IC

$$IR = \frac{\overline{IC}}{\text{std}(IC)}$$

- The results applies to individual factors, different combinations of factors
- They are different for different stock markets, style groups, sectors

Quantitative Equity Models

A Portfolio Appr

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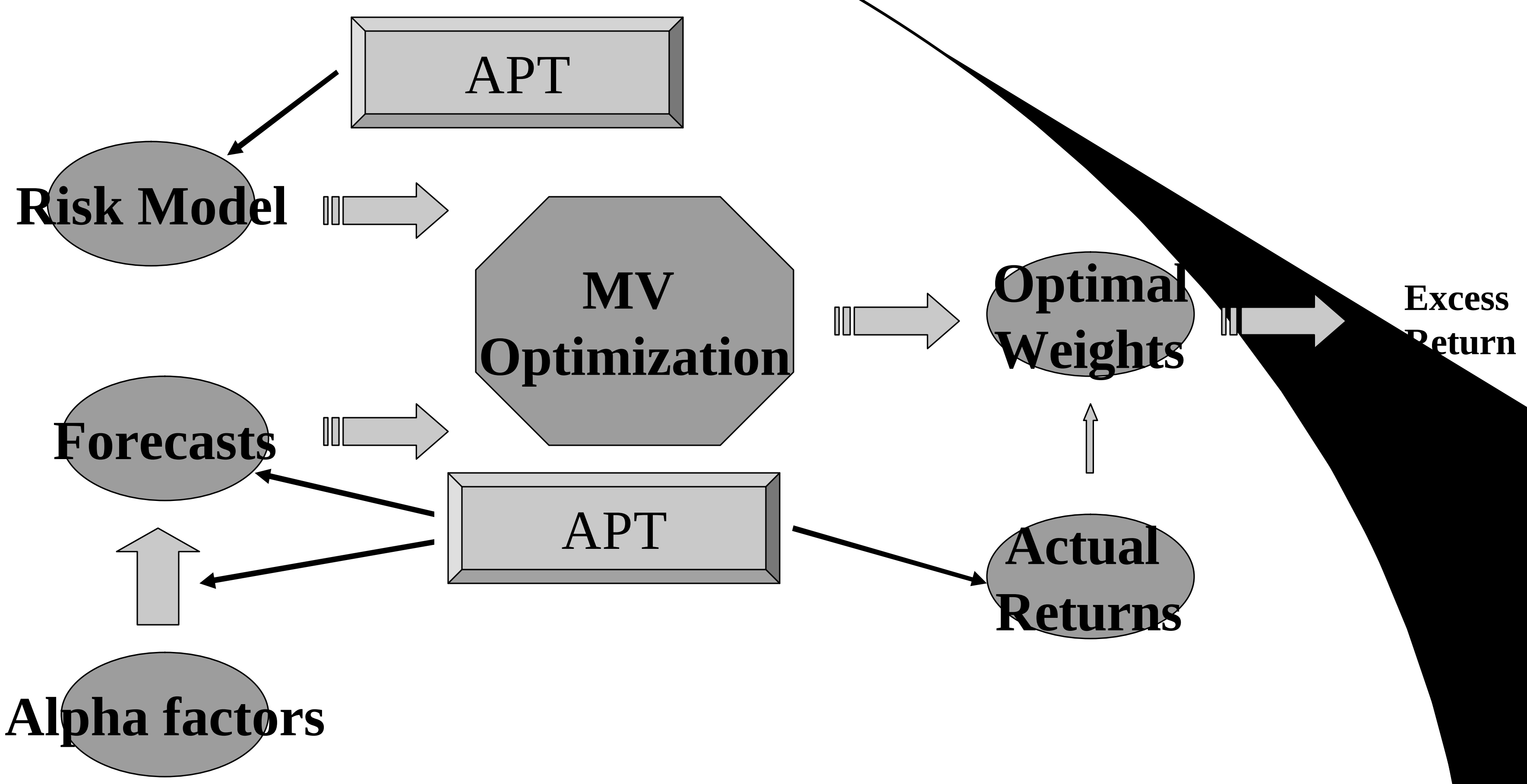
A New Approach

- New approach to build quantitative equity models
- The new approach is descriptive in nature
- We derive the new results based on th[illegible]proach
- It differs from the “normative” approach [illegible] FLOAM

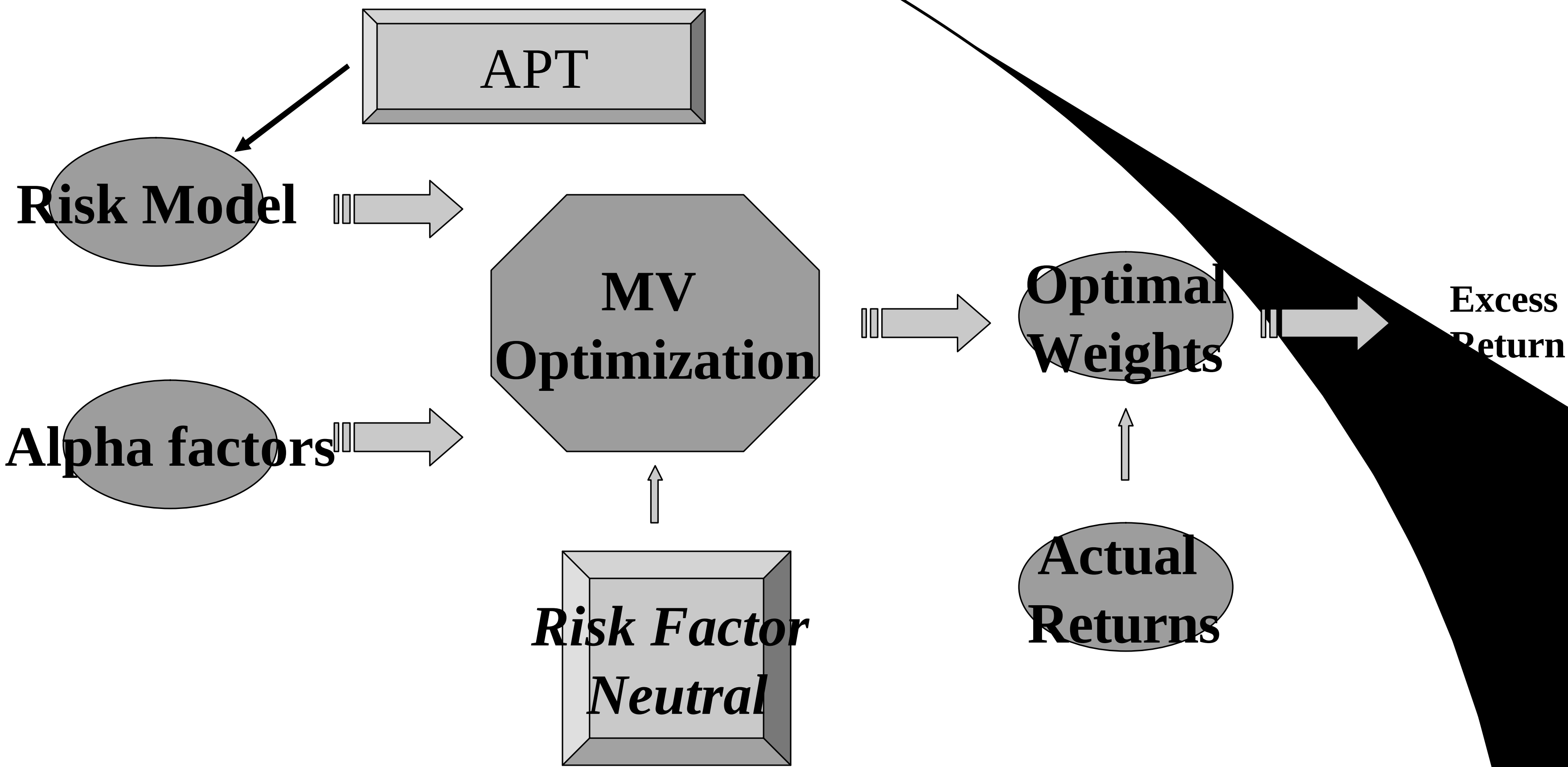
A Normative Approach



Normative Versus Descriptive

- Derivation of fundamental law depends on normative framework of APT
  - Fundamental risk models are often based on [illegible] framework
  - Expected excess returns of individual securities [illegible] assumed to follow APT: "beta" times alpha factor[illegible]
- Descriptive approach
  - Practitioners need risk model to do mean-variance optimization
  - But it is not necessary to APT will be true for each secur[illegible]
  - Based on "real" long-short portfolios

The New Approach



Details

- Select a risk model – BARRA, Northfield,APT ...
- Select a alpha factor – valuation, momentum, profitability, ...
- Select a fixed trading period – monthly, quarterly,..
- MV optimization chooses optimal long-short portfolio
  - With fixed target tracking error
  - With all risk factor neutralized
  - Exact solution exists for the optimization
- Compute single period alpha
- Compute multi-period statistics – average, standard deviation, IR, ...

Limitations and Extensions

- Focus on strategies exploiting only the stock-specific returns with no factor bets
  - Mathematical convenience
    - Exact solution for optimal weights
    - No need of historical factor return covariance matrices
  - Most quantitative strategies have minimal risks
- Analysis can be extended to strategies with factor
  - We can expect the same qualitative conclusion such as different strategy risk
  - The magnitudes of bias are not yet known

Single-period Results

- Optimal long-short portfolio weights

$$w_i = l^{-1}\frac{f_i - l_1 - l_2\boldsymbol{b}_{1i} - \cdots - l_{M+1}\boldsymbol{b}_{Mi}}{\boldsymbol{s}_i^2}$$

- Single-period excess return

$$\boldsymbol{a}_t = l_t^{-1}\sum_{i=1}^{N} R_{i,t}F_{i,t}$$

- Risk-adjust forecasts and risk-adjusted returns

$$F_i = \frac{f_i - l_1 - l_2\boldsymbol{b}_{1i} - \cdots - l_{M+1}\boldsymbol{b}_{Mi}}{\boldsymbol{s}_i} \quad R_i = \frac{r_i - k_1 - k_2\boldsymbol{b}_{1i} - \cdots - k_{M+1}\boldsymbol{b}_{M1i}}{\boldsymbol{s}_i}$$

- Single-period excess return in terms of IC and dispersions

$$\boldsymbol{a}_t = l_t^{-1}(N-1)IC_t \,\mathrm{dis}(\mathbf{R}_t)\,\mathrm{dis}(\mathbf{F}_t)$$

- Final results

$$\boldsymbol{a}_t \approx IC_t\sqrt{N}\,\boldsymbol{s}_{\mathrm{model}}\,\mathrm{dis}(\mathbf{R}_t)$$

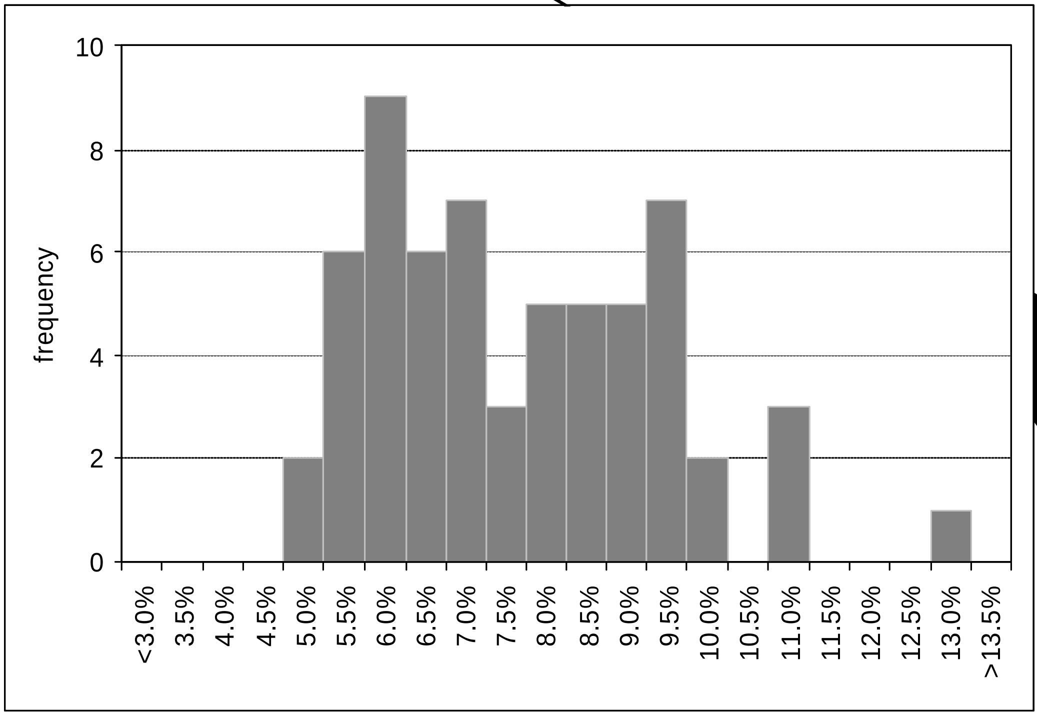
Implications

- The analysis is based on realistic long-short portfolios
- IC is correlation coefficient between risk-adjusted forecasts and risk-adjust returns
  - Not raw forecasts and raw returns
  - Not regressed forecasts
- We do not impose any relationship (linear) betw individual returns and individual forecasts, i.e., no
- For a consistent risk model, cross sectional dispersi risk-adjusted return should be close to unity
- Therefore $a_t \approx IC_t \sqrt{N} s_{\text{model}}$

Empirical Results

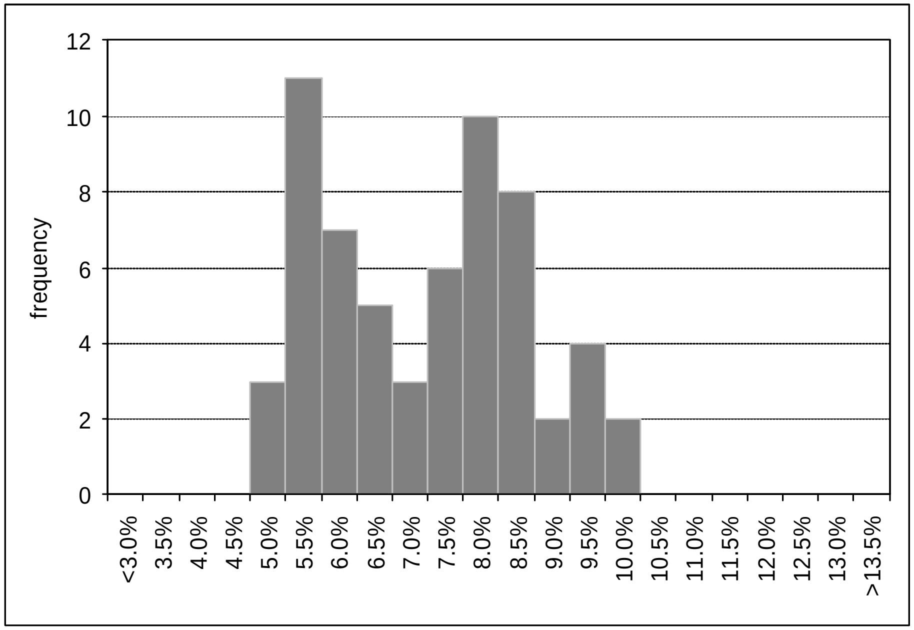
Higher Active R

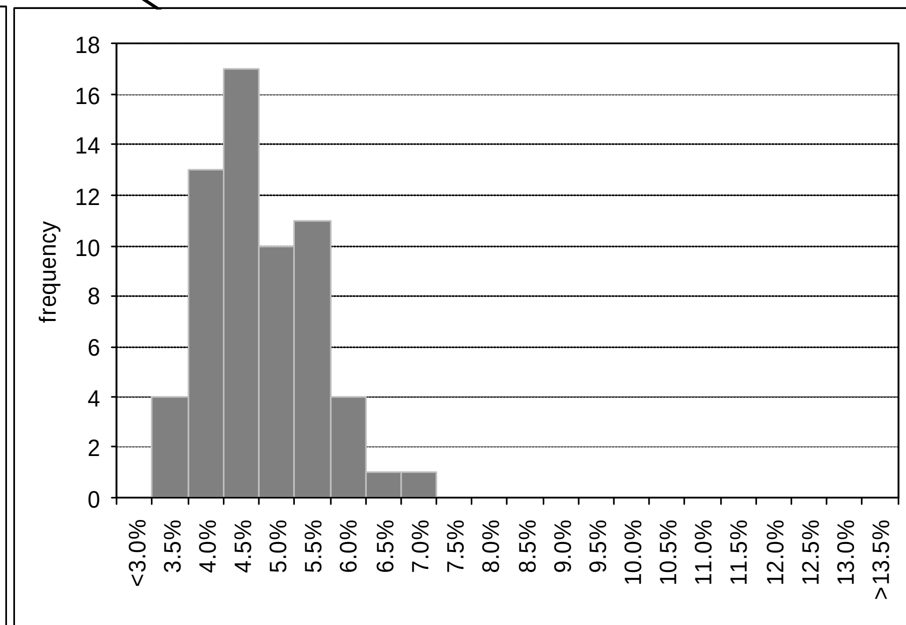
Ex Post Active Risk



Almost all alpha factors produce tracking error higher than the target of 5%. Some are significantly higher.

Persistence of Strategy Risk

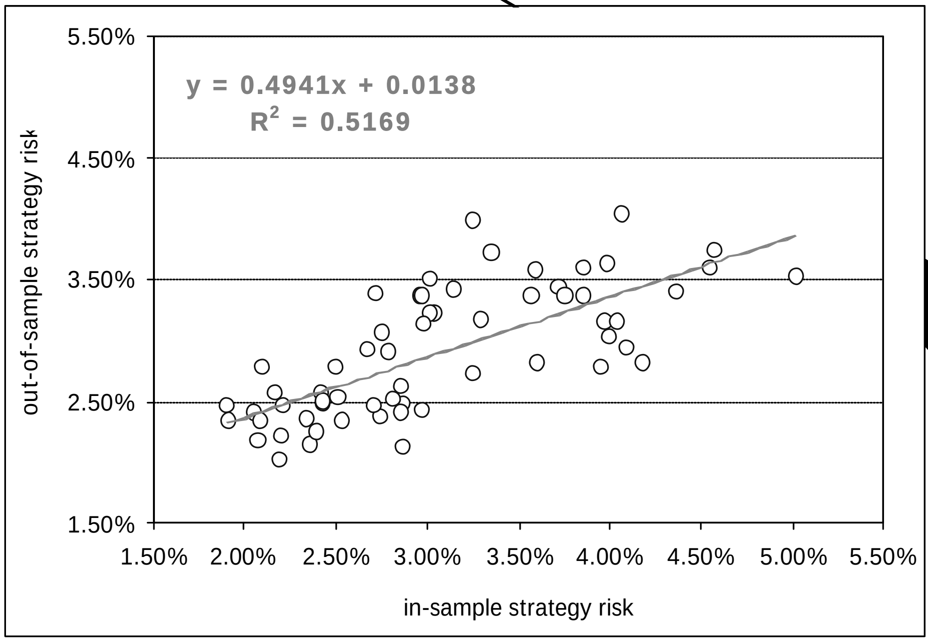




Second half active risk without adjustment

Second half active risk wit adjustment for strategy risk determined from the first half

Persistency of Strategy Risk



Strong positive correlation between two samples. Factors having higher (lower) strategy risk in the first half tends to higher (lower) strategy risk in the second half too

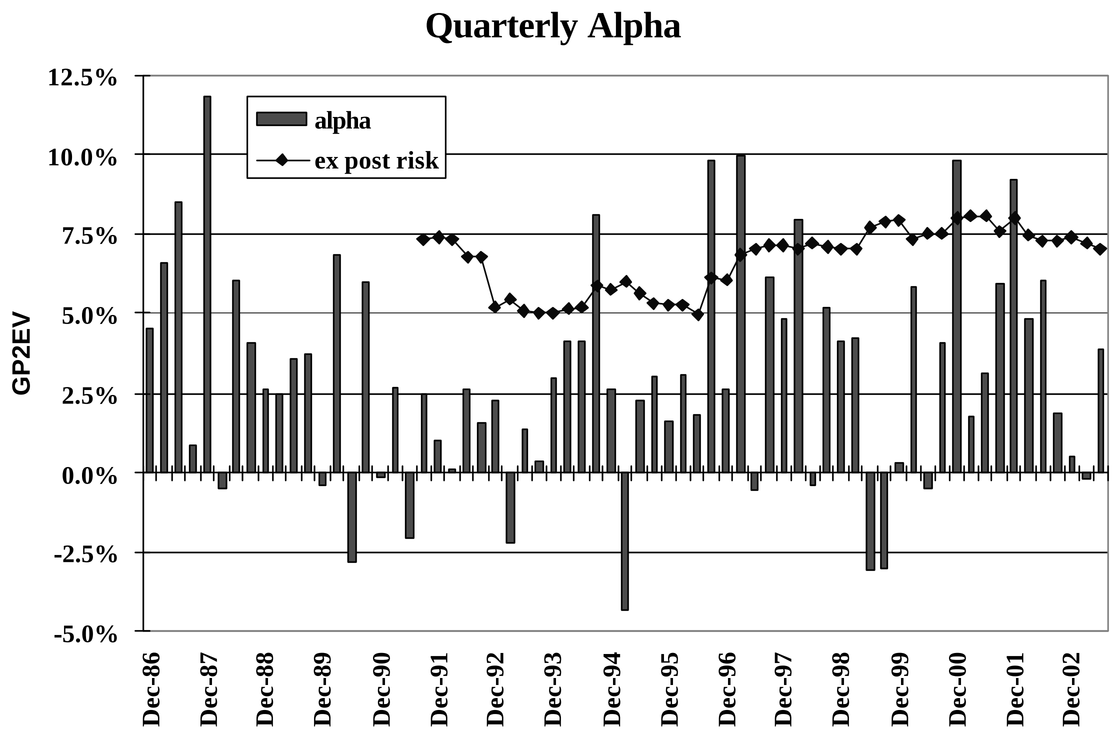
Significant Difference

- Two valuation factors
  - Gross profit to enterprise value (GP2EV)
  - Forward earnings yield based on IBES FY1 consensus forecast (E2P)

	Average Alpha	STD of Alpha	IR of Alpha	Average IC	STD of IC	IR of IC	Average dis(R)	Average N
GP2EV	6.2%	6.9%	0.90	2.4%	2.7%	0.91	1.01	2738
E2P	3.3%	8.7%	0.38	1.4%	3.4%	0.41	1.00	2487

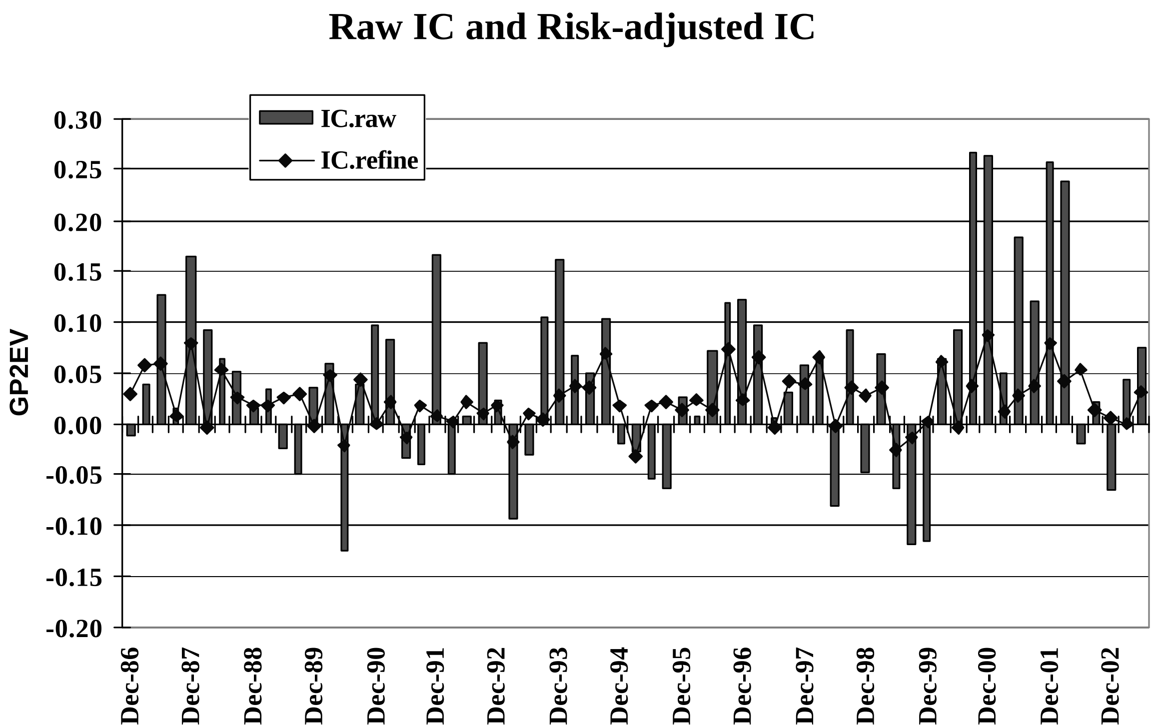
- F-test shows that the variances of IC are significant different at 5% level

A Factor Example



- A decent factor
- But the active risk is consistently above target

Information Coefficient



- The risk-adjusted IC is much more stable, indicating a better information ratio
- The raw IC carries risk-factor bias

Multi-factor Models

“Optimize” the IC

The Portfolio Approach

- Multiple forecasts for each N stocks
  - Previous research is unclear
  - Many practitioners resort to ad hoc weighting [illegible]hemes based on intuition or judgment
- Our approach provides a quantitative framew[illegible] combing multiple factors
  - For a multi-factor model, it is still true that $IR = \frac{\overline{IC}}{\text{std}(IC)}$
  - We maximize the IR to find the optimal linear combinat[illegible]
  - The problem is very similar to optimal allocation among multiple active managers
- Use optimal weights with caution and common sense

Important Inputs

- Each factor (manager)
  - Average IC, standard deviation of IC, IR
  - Analogous to expected alpha, active risk, IR
- Among factors (managers)
  - IC correlation: time series correlations between diffe
  - Analogous to correlations between excess returns of d managers
  - The correlations between different factors are not of prim importance
  - Factor correlation is not the same as IC correlation

Two-factor Case

- Combined IC of a single period

$$\mathrm{IC}_t = \frac{\mathrm{IC}_{q,t} \cdot w_q + \mathrm{IC}_{f,t} \cdot w_f}{\sqrt{1 - 2 w_q w_f (1 - r_t)}}$$

- The factor correlation is only a scale paramet[redacted]d when it is constant throughout time it has no effect o[redacted]
- Expected IR

$$\mathrm{IR} \approx \frac{\overline{\mathrm{IC}_t}}{std(\mathrm{IC}_t)} = \frac{\overline{IC}_{q,t} \cdot w_q + \overline{IC}_{f,t} \cdot w_f}{\sqrt{s_q^2 w_q^2 + 2 s_{f,q} w_f w_q + s_f^2 w_f^2}}$$

- The optimal weights can be found
- For multi-factor case, there is a matrix solution

Summary

- Portfolio approach: simplified backtest
- Strategy risk contributes additionally to active risk. It can be measured by standard deviation [illegible]f IC
- Use the proper definition of IC - the correla[illegible]n between risk-adjusted forecasts and risk-adjusted retur[illegible]t the raw IC
- IR is often lower than what FLOAM predicts, eve[illegible] there is no portfolio constraint and no model errors
- Select alpha factors based on average IC as well as standard deviation of IC
- Combine alpha factors based on the above as well as I[illegible] correlations