

# Momentum

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# Overview

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1. General Thoughts

2. Partial Moment Momentum

3. Distribution of Momentum Returns

# Topic 1: General Thoughts



Presented by: Stephen Satchell

# Momentum trading strategy



Cross sectional momentum (CSM) strategies are employed by buying previous winners and selling previous losers.

- Formation period, stocks are sorted based on their past performances.
- Holding period, long winners (best performing) while short losers (worst performing).
- Lag between formation and holding periods. Doing so can avoid short-term reversals.

# Profitability of momentum

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Momentum is profitable if:

1. Strong deterministic trend.
2. Some autocorrelation.
3. High risk positions sometimes have higher returns.
4. Reason 3 compatible with market efficiency.

# Quant; Profitability of momentum



We argue that CSM profitable when there are large differences in expected returns

1. Europe/Asia should be good for CSM (different countries & industries).
2. UK/US should be bad for CSM (homogeneous), but UK is in fact good.
3. Japan ??

# BF/Psychology; Profitability

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1. Practitioners of BF would say implicitly, Europeans, Brits, and Asians would have persistent psychological problems that do not correct.
2. Americans do not have these problems.

Quant explanation seems more plausible.

# Momentum, quant and psychology



1. When Quant (2007-2008?) became ugly, it re-emerged as behavioural (dates?).
2. Academics were hired to tell tales about psychological issues investors incurably had.
3. For example, Hong and Stein (1999) with different trader types under-reaction to overconfidence and overreaction to biased self-attribution.
4. A prospect-theoretical interpretation of momentum returns, see Menkhoff and Schmeling (2006).

# Topic 2

## Partial Moment Momentum

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Presented by: Yang Gao

## Why volatility matters



- Momentum profits benefit from persistent trends of the market which can be predicted by market volatility.

Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016) present evidence that scaling the weights of momentum portfolios increases the Sharpe ratio of the plain momentum strategy

# Why might upside versus downside risk matter



- Momentum crashes

*[Cooper, Gutierrez, and Hameed (2004), Chordia and Shivakumar (2002), Daniel and Moskowitz (2016)]*

- Short selling constraint in adverse times

*[Ali and Trombley (2006), Gao and Leung (2017)]*

- Investors risk preference

*[Daniel, Hirshleifer, and Subrahmanyam (1998) and Hong and Stein (1999)]*

[Further reading kindly see page 2-3 of the working paper. Available at SSRN (submission no.3019861) and Usyd BS finance discussion paper online.]

# Main findings

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- We also find that momentum profits benefit from persistent trends of the market which can be predicted by market volatility.
- We also find that partial moments-based (to be defined) momentum trading strategies significantly outperform plain momentum strategy.
- An explanation of this strong profitability is that momentum benefits from good versus bad risk.

# Data



- Monthly and daily US equity data over Jan 1927 to Dec 2016 sourced from the CRSP via WRDS. Our sample includes common stocks (CRSP share code 10 or 11) of all firms listed on NYSE, Amex and Nasdaq (CRSP exchange code 1, 2 or 3).
- We use the value-weighted index of all listed firms in the CRSP and 1-month Treasury bill rate as the proxy for the market portfolio and the risk-free rate, respectively. (Sourced from the Kenneth R. French Data Library)

# Partial moments



- Following the literature: Andersen, Bollerslev, Diebold & Ebens (2001), Barndorff-Nielson (2002) and Baruník, Kočenda & Vácha (2016)
- We define monthly realised variance  $RV$  as

$$RV_t = \sum_{i=1}^n r_{i,t}^2$$

- We define lower partial moment  $RPM^-$  and higher partial moment  $RPM^+$  as

$$RPM_t^- = \sum_{i=1}^n r_{i,t}^2 I(r_{i,t} < 0)$$
$$RPM_t^+ = \sum_{i=1}^n r_{i,t}^2 I(r_{i,t} \geq 0)$$

- There is an identity

$$RV_t = RPM_t^- + RPM_t^+$$

# Estimation



1. We use daily data for our different volatility estimators and use last month as our forecast for next month.
2. Alternatively, we also use a VAR(1) model based on  $RPM^+$  and  $RPM^-$ .
3. Method 2 does not do as well as method 1; old result hard to beat a random walk out of sample one period ahead.

# Adapted Sortino ratio (an improvement on Sharpe ratio)



- a performance measure better capturing the downside risk
- We define monthly realised variance  $RV$  as

$$\text{Adapted Sortino ratio} = \frac{\text{Excess Return}}{2 * \text{Downside SemiDeviation}}$$

where

$$\text{Excess Return}_t = R_t - \text{Desired Target Return}_t$$

*Downside SemiDeviation*

$$= \sqrt{\frac{\sum_{i=1}^N (\text{Excess Return}_i - \overline{\text{Excess Return}})^2}{N} I(\text{Excess Return}_i < 0)}$$

- Our adaptation differs from the standard Sortino (1994) ratio with target return equal to the riskless rate, as, in the event that mean returns are zero and  $RPM^- = RPM^+$ , we recover the Sharpe ratio.

# PM-based momentum strategies



These have the potential to better capture market trends and hopefully avoid huge losses during market rebounds using market decomposed variances.

## 1. Partial moment momentum strategy (PMM)

- Switching positions of the winner and loser portfolios during the holding periods depending upon current estimates of partial moments. ( $RPM^-$  &  $RPM^+$ )

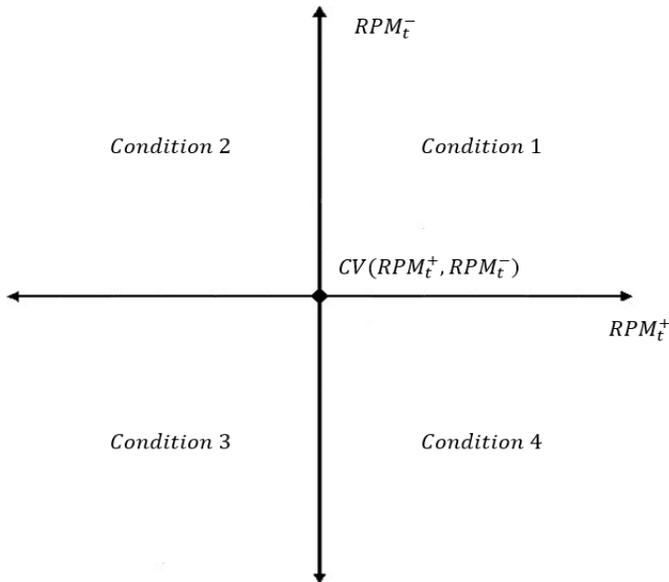
## 2. Extended partial moment-decomposed momentum strategy (PMD)

- Scaling the weights of winner and loser portfolios towards favourable/unfavourable volatility signals and holding an off-setting position in cash.

# Method 1, PMM construction



Condition 1	if $RPM_t^+ > CV(RPM_t^+)$	and if $RPM_t^- > CV(RPM_t^-)$
Condition 2	$RPM_t^+ < CV(RPM_t^+)$	$RPM_t^- > CV(RPM_t^-)$
Condition 3	$RPM_t^+ < CV(RPM_t^+)$	$RPM_t^- < CV(RPM_t^-)$
Condition 4	$RPM_t^+ > CV(RPM_t^+)$	$RPM_t^- < CV(RPM_t^-)$



**Two sets of CVs used for  $RPM_t^+$  and  $RPM_t^-$**

- 1. 10th percentile of  $RPM_t^+$  and 75th percentile of  $RPM_t^-$ .**
- 2. Medians of  $RPM_t^+$  and  $RPM_t^-$ .**

# PMM strategy analysis



In any given month  $t$ , conditions are classified based on partial moments in the previous month  $t-1$  due to one-month gap between the formation and holding periods;  $r_{w,t+1}, r_{l,t+1}, r_{f,t+1}$  represent returns of winners, losers and the risk-free asset in month  $t+1$ , respectively. Correspondingly, returns of momentum and contrarian strategies are  $r_{w,t+1} - r_{l,t+1}$  and  $r_{l,t+1} - r_{w,t+1}$ , respectively.

PMM Strategies	Condition 1	Condition 2	Condition 3	Condition 4
	Return	Return	Return	Return
PMM_S1	0	$r_{f,t+1} - r_{l,t+1}$	$r_{w,t+1} - r_{l,t+1}$	$r_{w,t+1} - r_{l,t+1}$
PMM_S2	$r_{l,t+1} - r_{w,t+1}$	$r_{f,t+1} - r_{l,t+1}$	$r_{w,t+1} - r_{l,t+1}$	$r_{w,t+1} - r_{l,t+1}$
PMM_S3	$r_{f,t+1} - r_{l,t+1}$	$r_{f,t+1} - r_{l,t+1}$	$r_{w,t+1} - r_{l,t+1}$	$r_{w,t+1} - r_{l,t+1}$
PMM_S4	0	$r_{f,t+1} - r_{l,t+1}$	$r_{w,t+1} - r_{l,t+1}$	$r_{w,t+1} - r_{f,t+1}$
PMM_S5	$r_{l,t+1} - r_{w,t+1}$	$r_{f,t+1} - r_{l,t+1}$	$r_{w,t+1} - r_{l,t+1}$	$r_{w,t+1} - r_{f,t+1}$
PMM_S6	$r_{f,t+1} - r_{l,t+1}$	$r_{f,t+1} - r_{l,t+1}$	$r_{w,t+1} - r_{l,t+1}$	$r_{w,t+1} - r_{f,t+1}$

# PMM performance (on a $6 \times 6$ basis, Whole sample period boundaries)



- Benchmark (M66 strategy) is a  $6 \times 6$  plain momentum strategy with one month gap between formation and holding periods.
- 10<sup>th</sup> percentile of  $RPM_t^+$  and 75<sup>th</sup> percentile of  $RPM_t^-$  as the critical values for upper and lower market partial moments
- Sharpe ratio, adapted Sortino ratio and returns are annualised.

# PMM performance (on a 6 × 6 basis, Whole sample period boundaries)



Strategy	Return	t-value	Sharpe ratio	Adapted Sortino ratio
<i>Panel A: P1, whole sample period: January 1927 to December 2016</i>				
M66	5.62	2.15 (**)	0.23	0.04
PMM_S1	5.48	3.84 (***)	0.40	0.08
PMM_S2	5.34	2.04 (**)	0.22	0.06
PMM_S3	2.35	0.64	0.07	-0.01
PMM_S4	14.74	7.08 (***)	0.75	0.36
PMM_S5	14.59	4.73 (***)	0.50	0.31
PMM_S6	11.36	2.80 (***)	0.29	0.09
<i>Panel B: P2, Jegadeesh and Titman (1993) period: January 1965 to December 1989</i>				
M66	12.03	3.59 (***)	0.72	0.15
PMM_S1	9.23	3.37 (***)	0.67	0.09
PMM_S2	6.49	1.95 (*)	0.39	-0.02
PMM_S3	10.62	2.71 (***)	0.54	0.08
PMM_S4	13.63	2.99 (***)	0.60	0.17
PMM_S5	10.79	2.19 (**)	0.44	0.10
PMM_S6	15.06	2.79 (***)	0.56	0.16
<i>Panel C: P3, market downturn: August 2007 to December 2012</i>				
M66	-8.98	-0.83	-0.35	-0.16
PMM_S1	5.06	1.40	0.60	0.34
PMM_S2	21.06	1.73 (*)	0.74	1.39
PMM_S3	-4.57	-0.29	-0.12	-0.07
PMM_S4	5.24	0.94	0.40	0.21
PMM_S5	21.26	1.64	0.70	1.04
PMM_S6	-4.41	-0.27	-0.12	-0.07
<i>Panel D: P4, era of turbulence: January 2000 to December 2016</i>				
M66	-0.92	-0.12	-0.03	-0.04
PMM_S1	4.20	1.46	0.35	0.13
PMM_S2	9.56	1.22	0.29	0.21
PMM_S3	-1.49	-0.16	-0.04	-0.04
PMM_S4	10.45	2.83 (***)	0.69	0.41
PMM_S5	16.11	1.92 (*)	0.47	0.38
PMM_S6	4.45	0.45	0.11	0.03

## Method 2, PMD strategy



- An extension of the dynamic volatility-based momentum strategy in Barroso and Santa-Clara (2015) where they do not differentiate between upside or downside risk.
- We extend by tilting our strategy long or short towards favourable/unfavourable volatility signals and holding an off-setting position in cash.

# PMD construction



The return of this strategy is denoted as

$$r_{p,t+1} = \varphi_1(RPM_t^+, RPM_t^-)r_{w,t+1} - \varphi_2(RPM_t^+, RPM_t^-)r_{l,t+1} \\ + (\varphi_2(RPM_t^+, RPM_t^-) - \varphi_1(RPM_t^+, RPM_t^-)) r_{f,t+1}$$

with the constraint

$$\varphi_1(RPM_t^+, RPM_t^-) + \varphi_2(RPM_t^+, RPM_t^-) = \frac{2\sigma_{tar}}{\sqrt{RV_t}}$$

where

$$\varphi_1(RPM_t^+, RPM_t^-) = \frac{2\sigma_{tar}}{\sqrt{RV_t}} \left( \frac{RPM_t^+}{RPM_t^+ + RPM_t^-} \right) \\ \varphi_2(RPM_t^+, RPM_t^-) = \frac{2\sigma_{tar}}{\sqrt{RV_t}} \left( \frac{RPM_t^-}{RPM_t^+ + RPM_t^-} \right)$$

- Zero-net positions
- If  $RPM_t^+ = RPM_t^-$ , then we have a conventional long-short portfolio with scaling  $\frac{\sigma_{tar}}{\sqrt{RV_t}}$  as in Barroso and Santa-Clara (2015), formula (5), page 115.

# PMD performance (on a 6 × 6 basis)



Strategy	Return	t-value	Sharpe ratio	Adapted Sortino ratio
<i>Panel A: P1, whole sample period: January 1927 to December 2016</i>				
M66	5.62	2.15 (**)	0.23	0.04
PMD	23.93	23.57 (***)	2.48	1.96
PMD_C	23.92	20.06 (***)	2.11	1.37
<i>Panel B: P2, Jegadeesh and Titman (1993) period: January 1965 to December 1989</i>				
M66	12.03	3.59 (***)	0.72	0.15
PMD	28.06	13.05 (***)	2.61	3.44
PMD_C	24.43	13.48 (***)	2.70	2.03
<i>Panel C: P3, market downturn: August 2007 to December 2012</i>				
M66	-8.98	-0.83	-0.35	-0.16
PMD	9.88	3.74 (***)	1.59	1.19
PMD_C	12.46	3.09 (***)	1.32	0.74
<i>Panel D: P4, era of turbulence: January 2000 to December 2016</i>				
M66	-0.92	-0.12	-0.03	-0.04
PMD	14.07	7.24 (***)	1.76	1.15
PMD_C	15.58	5.66 (***)	1.37	0.67

- Benchmark (M66 strategy) is a 6 × 6 plain momentum strategy with one month gap between formation and holding periods.
- PMD and PMD\_C represent an unconstrained PMD strategy and a 200% leverage-constrained PMD strategy, respectively.
- Sharpe ratio, adapted Sortino ratio and returns are annualised.

# Comparison of performance

(PMD strategies versus the Barroso and Santa-Clara (2015) volatility-scaled momentum strategy)



- Benchmark (WML strategy) is an  $11 \times 1$  plain momentum strategy with one month gap between formation and holding periods.
- PMD and PMD\_C represent an unconstrained PMD strategy and a 200% leverage-constrained PMD strategy, respectively.
- BSC represents the volatility-scaled momentum strategy constructed by Barroso and Santa-Clara (2015).
- Sharpe ratio, adapted Sortino ratio and returns are annualised.

# Comparison of performance (PMD strategies versus the Barroso and Santa-Clara (2015) volatility-scaled momentum strategy)



Strategy	Return	t-value	Sharpe ratio	Adapted Sortino ratio
<i>P1: whole sample period: January 1927 to December 2016</i>				
WML	19.05	5.58 (***)	0.59	0.24
BSC	17.30	8.82 (***)	0.94	1.04
PMD	28.16	24.28 (***)	2.56	2.16
PMD_C	27.47	21.47 (***)	2.26	1.51
<i>P2: Jegadeesh and Titman (1993): January 1965 to December 1989</i>				
WML	29.75	5.98 (***)	1.20	0.54
BSC	26.52	6.32 (***)	1.26	1.22
PMD	35.13	14.59 (***)	2.92	2.60
PMD_C	30.17	14.75 (***)	2.95	1.90
<i>P3: market downturn: August 2007 to December 2012</i>				
WML	1.46	0.08	0.03	0.01
BSC	11.83	1.88 (*)	0.80	0.98
PMD	13.87	3.94 (***)	1.68	0.98
PMD_C	19.06	3.14 (***)	1.34	0.66
<i>P4: era of turbulence: January 2000 to December 2016</i>				
WML	7.82	0.79	0.19	0.08
BSC	5.40	1.64	0.40	0.38
PMD	16.19	6.64 (***)	1.61	1.03
PMD_C	19.44	5.54 (***)	1.34	0.76

See BSC (2015) table 3, page 116.

# International Evidence



- Consistent with Daniel and Moskowitz (2016), we use the Asness, Moskowitz, and Pedersen (2013) “P3-P1” momentum portfolios obtained from the AQR Data Library. We refer to it as WML\_T.
- We report PMM and PMD performances for the UK, Japan and Europe along with the US over 1972–2016.

## AMP (2013) “P3-P1” momentum portfolios

- Data sourced from DataStream and XpressFeed Global.
- Tercile sort rather than decile sort.
- Large and liquid stocks only. The universe of stocks that account cumulatively for 90% of the total MV of the entire stock market. (similar to how MSCI defines for its global stock indices)
- Average no. of stocks: 724 (US), 147 (UK), 290 (Europe), and 471 (Japan).

# International Evidence



Strategy	UK			Continental Europe			Japan			US		
	Return	Sharpe ratio	Adapted Sortino ratio	Return	Sharpe ratio	Adapted Sortino ratio	Return	Sharpe ratio	Adapted Sortino ratio	Return	Sharpe ratio	Adapted Sortino ratio
<i>P1: whole sample period. For the UK and US, sample periods start from January 1972; for continental Europe and Japan, sample periods start from February 1974; for all four markets, sample periods end in December 2016.</i>												
WML_T	6.33(***)	0.39	0.06	6.75(***)	0.46	0.08	1.79	0.10	-0.10	4.79(**)	0.31	0.03
PMM_S1	7.36(***)	0.59	0.15	8.39(***)	0.75	0.23	2.95	0.20	-0.08	4.84(***)	0.45	0.09
PMM_S2	8.52(***)	0.52	0.18	10.00(***)	0.68	0.27	4.14	0.22	-0.02	4.88(**)	0.31	0.05
PMM_S3	5.26	0.23	0.01	6.26(**)	0.33	0.05	3.07	0.14	-0.05	3.75	0.21	-0.04
PMM_S4	0.21	0.01	-0.16	5.31(**)	0.35	0.02	1.52	0.09	-0.12	3.39	0.25	-0.06
PMM_S5	1.29	0.07	-0.12	6.87(**)	0.38	0.08	2.69	0.13	-0.07	3.43	0.19	-0.05
PMM_S6	-1.77	-0.07	-0.15	3.23	0.15	-0.05	1.63	0.07	-0.09	2.31	0.12	-0.08
PMD	14.47(***)	2.12	1.60	13.76(***)	2.18	1.44	13.58(***)	1.46	0.65	14.33(***)	2.32	1.69
PMD_C	17.19(***)	1.98	1.57	16.28(***)	2.14	1.66	14.39(***)	1.63	0.91	14.59(***)	2.09	1.14
<i>P2: market downturn: August 2007 to December 2012</i>												
WML_T	9.80	0.42	0.20	2.63	0.13	0.05	-2.11	-0.13	-0.12	0.51	0.03	-0.01
PMM_S1	3.94	0.36	0.17	6.86(*)	0.78	0.54	-0.63	-0.06	-0.09	5.96(*)	0.75	0.58
PMM_S2	-1.62	-0.07	-0.08	11.25	0.55	0.51	0.86	0.05	0.01	11.67	0.66	0.71
PMM_S3	10.00	0.30	0.17	6.54	0.19	0.13	2.68	0.13	0.07	0.82	0.03	0.00
PMM_S4	1.20	0.08	0.02	3.49	0.52	0.27	-0.95	-0.09	-0.09	2.51	0.30	0.12
PMM_S5	-4.23	-0.18	-0.14	7.75	0.40	0.39	0.54	0.03	-0.01	8.05	0.45	0.45
PMM_S6	7.11	0.20	0.11	3.18	0.10	0.06	2.36	0.12	0.06	-2.48	-0.10	-0.09
PMD	12.69(***)	1.91	2.22	10.25(***)	2.09	3.56	5.58(***)	1.48	1.44	9.77(***)	1.85	2.22
PMD_C	17.54(***)	1.86	1.60	19.46(***)	2.18	2.89	8.82(***)	1.43	1.18	12.80(***)	1.82	1.20

# Other robustness checks



All these tests reveal robust results.

## **PMM strategies with median boundaries**

- repeat all analyses using medians of  $RPM_t^+$  and  $RPM_t^-$  as boundaries

## **PMM & PMD on a WML (11 × 1) basis**

- In order to show the effectiveness of our PM-based momentum strategy compared with Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016).

## **Out-of-sample analysis**

- 1927 to 1999 as in-sample.

## **Dynamic PMM**

- Fit in-sample partial moments into a VAR(1) model and construct out-of-sample PMM using forecasted partial moments.

# Practical Matters

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1. No transaction costs.
2. No consideration of tradability and liquidity. If it is in CRSP, we use it. Maybe ok for factor but not for strategy. Methods used for international evidence might be more practical.

# Topic 3

## Distribution of CSM Returns

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Presented by: Stephen Satchell

# The Distribution of CSM returns



- Assumes returns on stocks are multivariate normal.
- Treats the problem as two-period, the formation and holding period.
- If two periods are independent and returns stationary, then markets efficient and high momentum returns a consequence of , presumably higher risk.
- Paper addresses what the distribution of CSM returns actually is.
- This has implications for payouts that CSM investors will experience.

# The Distribution of CSM returns



- Portfolios considered as m long, m short, universe n; Suppose  $n=2, m=1$ .
- If in formation period t:  $r_{1t} > r_{2t}$ , go long asset 1, short 2;  
If  $r_{1t} < r_{2t}$ , do opposite.

- $$Pdf(r_{mt+1}) = Pdf(r_{1t+1} - r_{2t+1} \text{ and } r_{1t} > r_{2t};) \\ + Pdf(r_{2t+1} - r_{1t+1} \text{ and } r_{1t} < r_{2t};)$$

- If markets are efficient; this becomes a mixture of univariate normals, in this case kurtotic and left skewed for plausible values:

$$Pdf(r_{mt+1}) = Pdf(r_{1t+1} - r_{2t+1})Prob(r_{1t} > r_{2t};) \\ + Pdf(r_{2t+1} - r_{1t+1})Prob(r_{1t} < r_{2t};)$$

- If markets not efficient (predictable) structure is more complicated but describable in terms of truncated bivariate normal distributions.

# Mixtures of Distributions



- What are they?
- Imagine you toss a coin first
- Then if heads you get  $N(0,1)$
- Then if tails you get  $N(5,10)$
- The resulting pdf is a mixture of normals

# The Distribution of CSM returns



- When are momentum expected returns positive?
- We do  $n=2$ ,  $m=1$  market efficient.
- Let  $P = Prob(r_{1t} > r_{2t})$ ; then

$$\begin{aligned} E(r_{mt+1}) &= (2P - 1)(E(r_{1t+1}) - E(r_{2t+1})) \\ &= (2P - 1)(\mu_{1t+1} - \mu_{2t+1}) \end{aligned}$$

$$P = \Phi\left(\frac{\mu_{1t} - \mu_{2t}}{\sqrt{\sigma_{1t}^2 - 2\rho\sigma_{1t}\sigma_{2t} + \sigma_{2t}^2}}\right) \text{ where } \Phi() \text{ is the normal distribution function}$$

- So you have to be able more than 50% of the time to pick the stock with the higher expected return, not surprising!

# The Distribution of CSM returns

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- Would high CSV be good/bad for CSM?
- This depends upon whether it is factor CSV(good) or idiosyncratic CSV(bad)
- We can see this from previous formula, factor CSV increases the numerator, other the denominator.

# The Distribution of CSM returns



- All the above generalise: for  $n$  assets,  $m$  long  $m$  short we need to know the number of possible rankings each of which contribute to the pdf of momentum. This number is  $\frac{n!}{(n-2m)!m!m!}$ .
- So if we want to investigate S&P500 long top 100 short bottom 100, we get a vast number but previous pdf generalises. We get  $\frac{n!}{(n-2m)!m!m!}$  terms in the pdf which involve univariate normals and truncated high dimensional multivariates.

# Result



- The result is that the density consists of a very large number of components.
- The case of 2 assets, it has 2 components (see slide 35).
- The case of N assets, it has  $\frac{n!}{(n-2m)!m!m!}$  components (see slide 37).

# Complexity



- The notion that expected utility maximisers take expected values over such a distribution becomes fanciful without access to modern MC.
- Consider  $\frac{500!}{(400)!50!50!}$ , this is huge, the age of the universe is 13,800,000,000 years the number of seconds in history is  $\exp(40)$  the number of pdfs the EU maximiser needs to consider is  $\exp(320)$ .

# Conclusions

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- We started arguing against BF.
- We derived the pdf of CSM which gives us some insights into more complex cases.
- We ended up questioning expected utility.



# Thanks!

*Any questions ?*

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## Appendix: Sharpe ratio (SR) for a long-short (LS)



- CAPM is a theory for long-only portfolios  $\mu_i - r_f = \beta_i(\mu_m - r_f)$  holds for stock  $i$ ,
- Suppose that  $w_i$  is the weight of stock  $i$  in the portfolio and  $\sum_{i=1}^n w_i = 0$ ,  $\sum_{i=1}^n \mu_i w_i = \mu_p$ , then CAPM for net zero portfolio is  $\mu_p = \beta_p(\mu_m - r_f)$ , where  $\beta_p = \sum_{i=1}^n \beta_i w_i$ , so that  $\mu_p - r_f = \beta_p(\mu_m - r_f) - r_f$  which does not seem right.
- Alternative; Suppose I view the SR as 100 per cent cash plus a long short, then  $SR = \frac{\mu_L - \mu_S}{sd(LS)}$  as the riskless rates cancel out, this is consistent with the CAPM for LS portfolios ( $\mu_p = \mu_L - \mu_S$ )