Professional Risk Managers' International Association

Maximizing RAROC: Turning the Risk Unit into a Profit Center

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presenter

- Leading Northfield’s ERM efforts for the last 18 years.
- Focus on cohesively integrating model approaches across asset classes, time horizons, and mandates (i.e. asset and liability, liquidity and solvency)
- Teaching Risk Management to graduate students
- New Frontiers of Risk Management Award, PRMIA, 2013
- Best Practitioner Research Award, American Real Estate Society, 2015

The Politics of Risk (*or the risk of politics*)

- By definition risky events are less likely than desirable outcomes from the perspective of a rational investor

- As long as the financial world goes mainstream (the more frequent case than not), the risk function seem to have a dubious value to the untrained eye

- With that, the costs and constraints that are associated with it are often seen as unnecessary

- Increased regulatory scrutiny have increased awareness of the need for a strong risk management function but have not transformed the view that “Risk” is a cost, not a profit center

Asset Managers Got a Boost

- In 1952, Harry Markowitz put down the fundamentals of Modern Portfolio Theory which positions risk and return side by side in the objective function:

  Utility = Expected Return - Risk Aversion * Risk

  where Risk is assumed to be the Variance of the portfolio

- This juxtaposition of risk and return immediately finds a place for the risk estimation in the portfolio construction process, and consequently, in the revenue potential of the asset manager: *"Taking Risk is How I make Return. How do I do this better?"*

- For a financial institution (e.g. a bank) where the revenue is not percentage of assets under management but a spread of interest or fees, this utility function is difficult to adopt

Risk Adjusted Return on Capital (RAROC)

- We will argue that the RAROC metric, even if unbeknownst to its original creators, can be adjusted to be a powerful objective function:
  - The adjusted RAROC objective function rivals MVO in potential benefits
  - It has distinct advantages over MVO when dealing with lumpy investments or those with non-normal distributions

- Consequently, we need a working definition. *Note that we have replaced Profit with Expected Value from the classic RAROC definition, as it is forward looking (and thus non-deterministic).*

$$adj.RAROC = \frac{Expected\ Value\ - Expected\ Loss}{Capital\ to\ Cover\ Unexpected\ Losses\ at\ Confidence\ \boldsymbol{\alpha}\%}$$

- **This is a simple but profound statement:** What is my profitability in relation to how much capital I have to invest, so that I am sure, with probability $\boldsymbol{\alpha}$ percent, that I will not go insolvent over the investment horizon of choice.

Time Travel To 2015

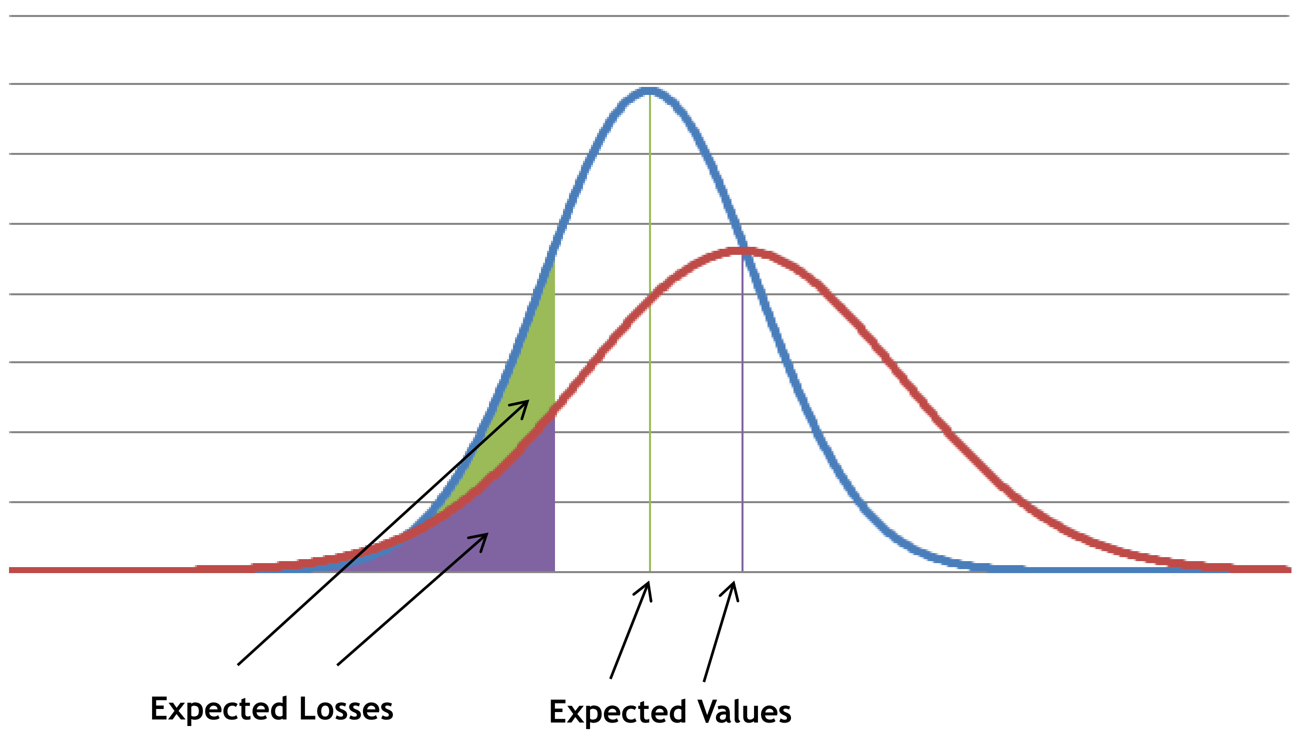
- The Objective of the Optimal Deal Flow For Illiquid Assets (ODFI) was developed (Belev, Gold, ARES award 2015).
  - It subjects a portfolio illiquid asset to optimization according to the numerator of the adjusted RAROC ratio
  - The only difference is that it takes the present value of the Expected Value

$$RAROC = \frac{Expected\ Value\ - Expected\ Loss}{Capital\ Needed\ to\ Cover\ Unexpected\ Losses\ at\ Confidence\ \alpha\%}$$

- It can be shown that this optimization also maximizes the whole ratio under a wide range of cases (e.g. under the normal distribution assumption, like MVO), under all confidences levels.

- With this presentation however, we will take a step further and optimize the whole ratio under the general case.

ODFI in a Picture



Clash of the Titans: Euler vs. Gauss

- Gauss has a frequent mention in finance due to the namesake synonym of the normal distribution which is assumed in MPT

- Euler contributed a bit lesser known finding, but in light of the results of this presentation, likely of a matching importance

- Euler's theorem of homogenous functions is an important part of the derivation of how to maximize adj. RAROC. If a function satisfies:

$$f(tx_1, \ldots, tx_k) = t^n f(x_1, \ldots, x_k)$$

then the following is true:

$$x_1 \frac{\partial f}{\partial x_1} + \cdots + x_k \frac{\partial f}{\partial x_k} = nf(x_1, \ldots, x_k)$$

Strategy for Deriving a RAROC Maximization Function

1. Assume an iterative process that embeds a modified version of the gradient method.

2. Represent the RAROC ratio objective in the form of a difference of the proportional change in the numerator and denominator.

3. Calculate derivatives of the numerator and denominator with respect to individuals positions. Use that as the incremental change basis for maximum ascent search path.

4. Use Euler's theorem to help break each divisor term that is used to calculate a proportional rather than absolute change within the difference expression per point 2 above.

5. Where necessary for the additive components in the expression (2), use Cornish Fischer expansion, which naturally further break down into moments of distributions

The RAROC objective As a Difference

- To increase a ratio, we have to increase the numerator proportionately more than the denominator:

$$\frac{A}{B}\frac{(1+k+s)}{(1+k)} = \frac{A}{B} + \frac{A}{B}s > \frac{A}{B}$$

- Consequently, at each step of our iterative process we can target to maximize the difference:

$$RAROC\ Objective = \frac{\partial A}{A} - \frac{\partial B}{B}$$

Where:

$$A = Expected\ Profit\ - Expected\ Cost$$

$$B\ = Economic\ Capital = VaR(\alpha)\ - Expected\ Cost$$

Estimating Derivatives of Terms

- Expected Value E[V]

$$E[V] = \sum_{k=1}^{N\ outcomes} p_k \sum_{i}^{M\ positions} x_i V_{i,k}$$

$$\frac{\partial V}{\partial x_i} = \sum_{k=1}^{N\ outcomes} p_k V_{i,k} = E[V_i]$$

- Expected Loss
  - Can be estimated similarly to expected value but as a conditional expected value, given that losses diversify across positions
  - Can also be estimated empirically by taking small increments of the position size and recalculating the resulting increment of the Expected Loss

Estimating Derivatives of Terms (cont'd)

- VaR – as a start we will note that central moments have a high importance to the VaR calculation through Cornish Fisher expansion
- That is why we will be interested in the calculation of the first derivative of the central moment. It can be shown that a moment of order j has the following first derivative with respect to a position $\mathbf{x_s}$:

$$E[(V - E[V])^j]' = j \sum_{k=1}^{N\,outcomes} p_k \Big(\sum_{i=1}^{M\,positions} x_i V_{i,k} - E[V] \Big)^{j-1} * (V_s - E[V_s])$$

- This can be though of the "covariance" of the value of the particular position with the one degree lower order moment of the overall portfolio: **this reflects the position's diversification properties !**

Cornish-Fisher Expansion of VaR

- Based on cumulants which are directly linked to central moments:

$$k_1 = \mu_1 \quad k_2 = \mu_2 \quad k_3 = \mu_3$$

$$k_4 = \mu_4 - \mu_2^2 \qquad k_5 = \mu_5 - 10\mu_3\mu_2$$

- An approximation using the first five moments/cumulants:

$$VaR = VaR_{normal} + Constant1 * k_3 + Constant2 * k_4 - Constant3 * k_3^2 + \text{Constant4} * k_5 + Constant5 * k_3 k_4 + Constant6 * k_3^3$$

- Partial derivative of VaR with respect to any position can be calculated using the derivatives of the additive terms with respect to the same position

Allocating Terms to Positions in Divisors

- We can utilize Euler's theorem to calculate divisor terms A and B. Note that all three types of components Expected Profit, Expected Cost, and VaR, when presented in momentary terms ($,¥,€,£, etc.) are homogenous functions.

- Let's take VaR for example and assume two position weights x1 and x2:

$$VaR = x_1 \frac{\partial VaR}{\partial x_1} + x_2 \frac{\partial VaR}{\partial x_2}$$

- Note: Derivatives of VaR with respect to positons can also be calculated empirically in addition to using Cornish Fisher expansion.

Maximize the Objective

- Putting it all together we get:

$$Local\ Objective = \frac{\frac{\partial E[Value]}{\partial x} - \frac{\partial E[Loss]}{\partial x}}{Euler\ Expansion\ of\ (E[Value] - E[Loss])}$$

$$- \frac{\frac{\partial (Cornish\ Fisher\ VaR)}{\partial x} - \frac{\partial E[Loss]}{\partial x}}{Euler\ Expansion\ of\ (VaR - E[Loss])}$$

- The maximum ascent path is chosen in the direction of the largest value of the local objective at the particular iteration

- Solution can be made more efficient if adj. RAROC ratio is solved to "1" and then apply maximization procedure

Application of the Objective Form

- It can be embedded in a computer algorithm that optimizes portfolios of positions at the margin
  - Fits the need of liquid asset managers that have requirements beyond MVO
  - Also reflect any acquisition of new assets or projects in an organization that are finely divisible
- It can also be applied to determination of choice of large scale (lumpy) projects where the steps and the options in choosing the optimal portfolio of project are much less
  - This reflects strategic decisions of business operations by type, geography, or manager mandate
  - It would also serve adding larger loans positions to the bank or insurance portfolio (i.e. a commercial real estate mortgage)

Does our Objective Reflect Risk Aversion

- Assuming log normal returns in his Discretionary Wealth Hypothesis, Jarrod Wilcox (2003), derived an expression for the risk aversion parameter that equates it with half of the leverage ratio of the investor

- Our RAROC objective does not need an explicit risk aversion parameter. Risk aversion is embedded in the nature of its calculation
  - **The higher the leverage, the higher the Expected loss and Economic Capital and hence the higher the drag on profitability**
  - **We can even impose additional Expected Loss and Economic Capital hurdles by including in them planned future cash flow outlays that can be interpreted as liabilities, which may or may not be legal obligations**
  - **One subtle but profound aspect of the chosen level of risk aversion in an optimization is the probability with which we want to avoid going bankrupt, that is determined by the α parameter in the denominator of the RAROC ratio. A more risk averse investor may pick to optimize under a higher α than a more risk prone investor.**

Summary

- Using a RAROC maximization procedure, the risk departments can be an active participant in promoting superior risk-adjusted performance

- A risk department's contribution can be measured by the difference of the average RAROCs of individual investments and projects given to its discretion vs. the optimal RAROC of the portfolio of the resulting investments/project.

- The approach is intuitive, tractable, and addresses a very wide array of real world statistical distributions determined by economic regimes and position types. It also reflects directly the appropriate risk aversion without the need of subjectivity or obscurity about what risk aversion actually means.

Thank you for viewing this session!

Please direct any follow up questions to:
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