MARKET IMPLIED GDP

HARRIS NTANTANIS & LARRY POHLMAN, PHD

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NORTHFIELD’S 30TH ANNUAL RESEARCH CONFERENCE
OUTLINE

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BACKGROUND

• IN 2012, INVESTORS JOINED GREECE ON A SWAP AGREEMENT (PSI)

• NEW BONDS, SHORT-TERM NOTES AND A WARRANT FOR THE EXCHANGE OF THE PREVIOUS DEBT

• THE WARRANTS ARE A PUBLIC LISTED SECURITY WITH SOME SPECIAL FEATURES:
  1. MULTIPLE ANNUAL EXERCISE DATES (TREAT IT LIKE A SERIES OF CAPLETS)
  2. CASH-OR-NOTHING UP-AND-IN DUAL STRIKE LOOK BACK BARRIER
  3. BARRIERS RESET ON A ANNUAL BASIS UNTIL 2020, STABLE THEREAFTER
  4. PAYMENT IS CAPPED TO UP 1% OF THE NOTIONAL AMOUNT
  5. LIFETIME: 2012-2042
Barrier levels for warrant to be "in the money"

<table>
<thead>
<tr>
<th>Reference Year</th>
<th>Reference Nominal GDP in billion euros</th>
<th>Reference Real GDP Growth Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>2014</td>
<td>210.10</td>
<td>2.35%</td>
</tr>
<tr>
<td>2015</td>
<td>217.90</td>
<td>2.90%</td>
</tr>
<tr>
<td>2016</td>
<td>226.35</td>
<td>2.85%</td>
</tr>
<tr>
<td>2017</td>
<td>235.72</td>
<td>2.80%</td>
</tr>
<tr>
<td>2018</td>
<td>245.47</td>
<td>2.60%</td>
</tr>
<tr>
<td>2019</td>
<td>255.88</td>
<td>2.50%</td>
</tr>
<tr>
<td>2020-2041</td>
<td>266.47</td>
<td>2.00%</td>
</tr>
</tbody>
</table>

BARRIERS FOR THE PAYMENT

When both GDP and GDPR are above these thresholds a payment takes place in the Q3 of the next year. GDP level and real GDP growth rate are published by Eurostat.
RESEARCH QUESTION

SINCE WARRANTS ARE LISTED, DO THEIR PRICES HAVE INFORMATION ABOUT GDP?

Two approaches:

• Regression pricing modeling

• Option Pricing
DATA

- GDP NOMINAL AND REAL GDP GROWTH RATE
- WARRANT QUARTERLY AVERAGE OF DAILY PRICES
- QUARTERLY FROM 2012 TO PRESENT
- BASED ON THE BARRIERS WE ARE CALCULATING TWO MORE VARIABLES (SPREAD LEVEL AND RATE)

In sample Warrant Prices, GDP, GDPR, GDPS & GDPSR
RESEARCH APPROACH

• REGRESSION BASED

\[ GDP_t \equiv a + b_1 \times WP_t \]

\[ GDPR_t \equiv a + b_1 \times WP_t \]
OPTION PRICING MODELS

BLACK SCHOLES

\[ c = FN(d_1) - KN(d_2)e^{-rt} \]

BACHELIER (NEGATIVE VARIABLES)

\[ c = (F - K)N(d) + \sigma \sqrt{t} n(d) \]

REINER & RUBENSTEIN (BARRIER OPTIONS)

\[ c = e^{-rt} KN(d) \]

\[ d_1 = \frac{\ln(F/K) + t\sigma^2 / 2}{\sigma\sqrt{t}} \]

\[ d = \frac{F - K}{\sigma\sqrt{t}} \]
OPTION PRICING RESEARCH APPROACH

- WARRANT PRICING, HAYMEN & KAT

\[ c_{i,j} = e^{-rt_{i,j}} K_j M(d^l_{i,j}, d^r_{i,j}; \rho) \]
\[ C_j = \sum_{i=j}^{n} c_{i,j} \]
\[ d^l_{i,j} = \frac{(S^l_{i,j} - X^l_{i,j}) - t_{i,j} (\sigma^l_j)^2 / 2}{\sigma^l_j \sqrt{t_{i,j}}} \]
\[ S^l_{i+1,j} = S^l_i \ast \sigma^l_j \]
\[ d^r_{i,j} = \frac{(S^r_{i,j} - X^r_{i,j}) - t_{i,j} (\sigma^r_j)^2 / 2}{\sigma^r_j \sqrt{t_{i,j}}} \]
\[ S^r_{i+1,j} = S^r_i + \sigma^r_j \]
ESTIMATION

• CONVENTIONAL

\[ GDP_t = a^w + b_1^w * WP_t + b_2^w * \exp(WP_t) + \varepsilon_t \]  \text{(ESTIMATE)}

\[ WIGDP_t \equiv a^w + b_1^w * WP_t + b_2^w * \exp(WP_t) \]  \text{(FORECAST)}

• OPTION IMPLIED

FOR EVERY QUARTER I WE CALCULATE THE VALUES OF \( S^L \) AND \( S^R \) BY SOLVING SIMULTANEOUSLY FOR THE \( \sigma^L_j \) AND \( \sigma^R_j \) THAT EQUATES THE CAP \( C_j \) TO WARRANT PRICE. THE OPTION IMPLIED GDP (OIGDP) AND GDPR (OIGDPR) EQUAL THE VALUES OF \( S^L_{1,j} \) AND \( S^R_{1,j} \).
IMPROVING FORECASTS

- $GDP_t = a + b_1 * GDP_{t-1} + \ldots + \varepsilon_t$  
  (BASIC AUTOREGRESSION EQUATION)

- $GDP_t = a + b_1 * GDP_{t-1} + b_2 * WIGDP_{t-1} + \varepsilon_t$  
  (CONVENTIONAL FORECAST)

- $GDP_t = a + b_1 * GDP_{t-1} + b_2 * OIGDP_{t-1} + \varepsilon_t$  
  (OPTION IMPLIED FORECAST)
### RESULTS

<table>
<thead>
<tr>
<th>Variables</th>
<th>Warrant Implied</th>
<th>Option Implied</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>1.00 (0.00)</td>
<td>0.96 (0.00)</td>
</tr>
<tr>
<td>GDPR</td>
<td>1.00 (0.00)</td>
<td>0.81 (0.00)</td>
</tr>
<tr>
<td>GDPS</td>
<td>1.00 (0.00)</td>
<td>1.13 (0.00)</td>
</tr>
<tr>
<td>GDPSPR</td>
<td>1.00 (0.00)</td>
<td>0.96 (0.00)</td>
</tr>
</tbody>
</table>
CONCLUSIONS

Both Conventional and Option Implied approaches

Improve GDP forecasts
QUESTIONS?