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Dissecting "Duration Times Spread"

Dan diBartolomeo

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Introduction

- In recent years, the "Duration Times Spread" (DTS) methodology has become the most commonly used approach to estimate the risk of bond portfolios. DTS was first formalized by Ben Dor, et. al. (*Journal of Portfolio Management*, 2007). Put simply it suggests that the volatility of the portion of bond yields associated with credit and liquidity risks (i.e. OAS) is linearly related to the magnitude of the spread itself.

- In early 2019, Northfield will be introducing a fixed income factor risk model that will use the DTS concept but will also incorporate a number of important enhancements to address what we believe are key weaknesses in the DTS approach.

DTS Basics

- It should be intuitive that OAS spreads should be non-negative. As such, we need an appropriate distributional assumption (e.g. lognormal, gamma, chi-square)
- The DTS method is a simple form of conditional risk model where the expected value of the volatility of a factor is a unconditional estimate (i.e. historical) adjusted by some scalar for current conditions.
  - Assume that we have a category of bonds (e.g. AA rated Euro denominated industrial corporate bonds) which over the past 120 months have had an average spread of 200 basis points, with a spread volatility (standard deviation) of 40 basis points.
  - Assume the spread rises to 300 basis points. Under DTS we assume that the expected spread will be 60 basis points since this is 20% of the current 300 basis point spread, just as 40 basis points was 20% of the 200 basis point historical average spread.

DTS Fits the Available Data

- While this construct fits empirical data very well there are a number of limitations to this approach of which investors must be aware. The first matter of interest is that arithmetic value of the "duration times spread" is exactly the *maximum amount by which a bond portfolio can increase in value* due to the portfolio becoming riskless with respect to creditworthiness and liquidity.
- Most investors would find the idea of measuring risk by how much a portfolio can increase in value unintuitive.   Implicit in the idea that a measure of "upside" should be considered an estimate of risk is that the *distribution of credit related returns for a bond portfolio is assumed to be symmetric.*
- We would clearly not expect the returns of a single bond issue subject to default risk to be symmetric. Under the Central Limit Theorem, we can justify not only the expectation of symmetry but also normality for the distribution of returns.

But Does the CLT Apply?

- However, the requirements of the CLT are that we are summing a large number of independent return distributions (i.e. the returns of individual bonds). To the extent that the creditworthiness and liquidity of the bonds in a particular portfolio are likely to be correlated rather than independent there must be a degree of portfolio concentration such that the CLT requirements are not met and *the expectation of symmetry must fail.*
- This situation is similar to the assumptions made under the Gaussian Copula process for debt securitizations by Li (*Journal of Fixed Income*, 2000) which were subject to material criticism in the "Global Financial Crisis" period.
- The CLT assumption must fail for the case of concentrated portfolios or for in crisis periods when bond defaults (or extreme spread movements). *In some sense, the assumptions of the model are most likely to fail in crises when a risk model is most needed.*

Spread Boundary Conditions

- Another limitation of DTS that as the creditworthiness of a bond falls investors *must eventually reach a point where there is a high degree of certainty that a particular bond will default.*
  - In such a case, the value of the spread should increase (the expected loss is increasing due to the likely default), but the volatility of the spread should decrease as bond approaches default.
  - This situation contradicts the basic DTS premise that the volatility of spreads should be linearly (positively) related to spread levels.
- A comparable limitation has been argued for other end of the spectrum.
  - Under DTS, spread volatility should decline with spread so that bonds are perceived to be almost riskless should have the lowest spread volatility.
  - To the extent that there are a very small number of issuers whose bonds are likely be considered riskless by all global investors, the requirements of the Central Limit Theorem are not met, so the symmetry argument would not apply.

A Paradox

- The original DTS paper also suggests that the idiosyncratic risk of bonds varies proportionately with spread.
- If we are measuring spread at the bond cohort level, this seems immediately unintuitive. If the risks are idiosyncratic within a group of bonds, how can we justify scaling by a common value across different issuers?
- If we apply DTS at the individual issuer or bond level, we have a different problem. If we scale idiosyncratic risks by relative spreads, we are implicitly assuming that the proportion of risk coming from common factors and idiosyncratic sources is fixed. Again this seems counterintuitive.
- Some existing DTS models have introduced some form of “default model” overlay to try to address this issue.

A Grain of Salt

- Finally, we would note that all empirical research on fixed income securities may be limited in the sense that the widely disseminated prices for bonds are derived *estimates* typically from "matrix pricing" models with limited amounts of actual trade data as input.

- As such, the in-sample explanatory power of any such model may be materially overstated because the matrix pricing model and the risk model simply have common underlying assumptions.

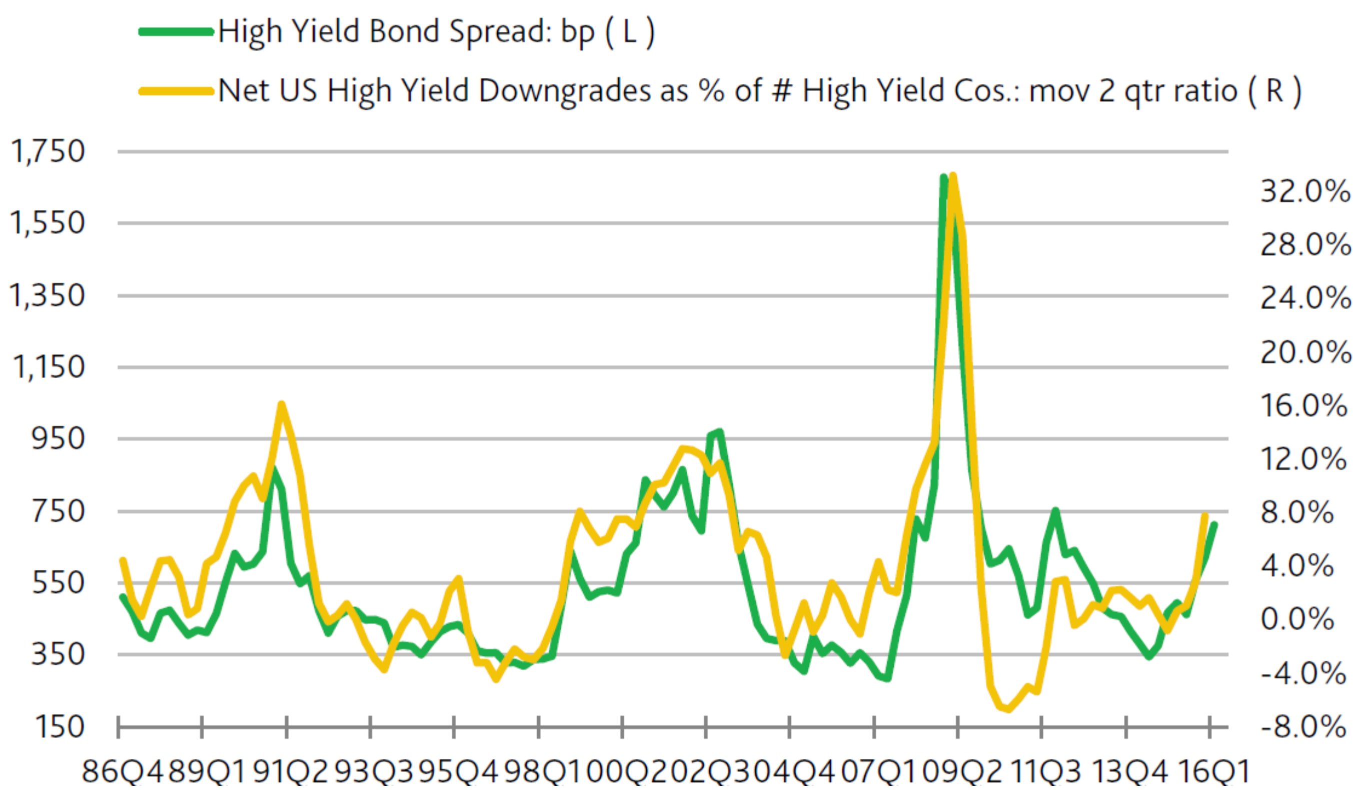
- To the extent that bonds are still traded in a "dealer" market, the impact of transaction costs is unclear. Exactly what the "spread" is for a given bond in a given transaction is not easily defined.
  - For background on bond trading costs see this presentation from our 2017 London seminar, http://www.northinfo.com/Documents/760.pdf.

Relating Spreads to Actual Default Risk

- There are two conflicting concepts of what credit risk actually is. The classic definition has to do with the likelihood that a given fixed income instrument will default (PD), and the expected severity of economic loss in the event of a default (LGD). The focus is on the "tail risk" (negative skew in the return distribution) associated with the default event
- Other market participants prefer to think of a given fixed income instrument as offering a yield spread above a comparable duration riskless instrument. Investors think of credit risk as the volatility of the credit related yield spread and impact on the market value of an instrument (conditional on the duration). If investors are not risk-neutral, the spread must compensate for expected loss (PD*LGD), plus provide a risk premium to for the uncertainty of spreads.
- The new model will use the credit analytics from our EE model to improve the estimated credit spread volatility and correlation.

Spread/Moody's Rating Relationship is Slow

Figure 2: Recent High-Yield Bond Spread Is Somewhat Wider than What Is Predicted by the Net High-Yield Downgrades of 2015's Second Half



Basic Contingent Claims Literature

- Merton (1974) poses the equity of a firm as a European call option on the firm's assets, with a strike price equal to the face value of the firm's debt
  - Alternatively, lenders are short a put on the firm assets
  - Default can occur only at debt maturity

- Black and Cox (1976) provide a "first passage" model
  - Default can occur before debt maturity
  - Firm extinction is assumed if asset values hit a boundary value (i.e. specified by bond covenants)

- Leland (1994) and Leland and Toft (1996)
  - Account for the tax deductibility of interest payments and costs of bankruptcy. Estimate strike price "boundary" where firm equity value is maximized subject to bankruptcy

Estimating PD and LGD

- Underlying is the firm's assets with asset volatility determined from the equity factor model
  - *How volatile would a firm's stock be if the firm had no debt?*
  - *This is the volatility of the assets*

- Include a term structure of interest rates so that as the implied expiration date moves around, the interest rate changes appropriately

- You can solve numerically for the "implied expiration date" of the option that equates the option values to the stock price
  - Market implied expected life of the firm, which is a transform of PD
  - See diBartolomeo (*Journal of Investing*, 2010).
  - See Yaksick (1998) for numerical methods for evaluating a perpetual American option

Defining the Default Risk in Corporate Bonds

- PD is the "percent moneyness" of the put option

- One approach to approximate "Loss Given Default" without using calculus is

$$LGD = (-(T-B)/B) * (\Delta_p / \Delta_c)$$

T is the value of the bond if it were riskless

B is the market value of the bond

Δ_p = delta of the put option

Δ_c = delta of the call option

Volatility in the values of PD and LGD at the individual security level is largely driven by the common change in the volatility of firm equity, resulting in positive correlation.

Extension to Sovereign Debt

- Bodie, Gray, Merton (2005, 2007)
  - The paper provides a complex system of theoretical balance sheet relationships among three types of entities: the Corporate Sector, the Financial Sector including Central Banks, and Sovereign Governments
- The interrelationships between sectors are modeled as a set of put and call options among the players
  - The government has a call on corporate assets (taxes)
  - The banks have a call on the government (bailouts)
  - *A key attribute (asset) of some but not all governments is a monopoly authority on the printing of money*
- Belev and diBartolomeo (2013) extends the model to include joint default risk of governments and banking systems
  - Winner of the PRMIA 2013 Award for "New Frontiers in Risk Management"

Basic Analytical Model of Spreads

- Our model assumes a mean/variance efficient investor

$$OAS_t = E[\text{credit losses} + \text{trading costs}] + \text{risk premium}$$

$$OAS_t = (PD_t*LGD_t)+(C*T)+L_t(var((PD_t*LGD_t)+k_t))$$

$$OAS_t = (PD_t*LGD_t)+(C*T)+ var(L_t*PD_t*LGD_t) + var(L_t*k_t)$$

C = estimated cost to trade bond

T = decimal annual portfolio turnover

var = variance statistical operator

E [] = expectations operator

L = investor risk aversion

k = illiquidity risk measure (no counterparty available)

Variance of a Product

$OAS_t = (PD_t*LGD_t)+(C*T)+ var(L_t*PD_t*LGD_t) + var(L_t*k_t)$

- Calculation of the term in green requires finding the variance of a product of two variables. Calculation of the term in red is the variance of three terms. If X and Y are independent, the variance of (XY) is

  $Var(XY) = var(X)*var(Y) + var(X) * E(Y)^2 + var(Y) * E(X)^2$

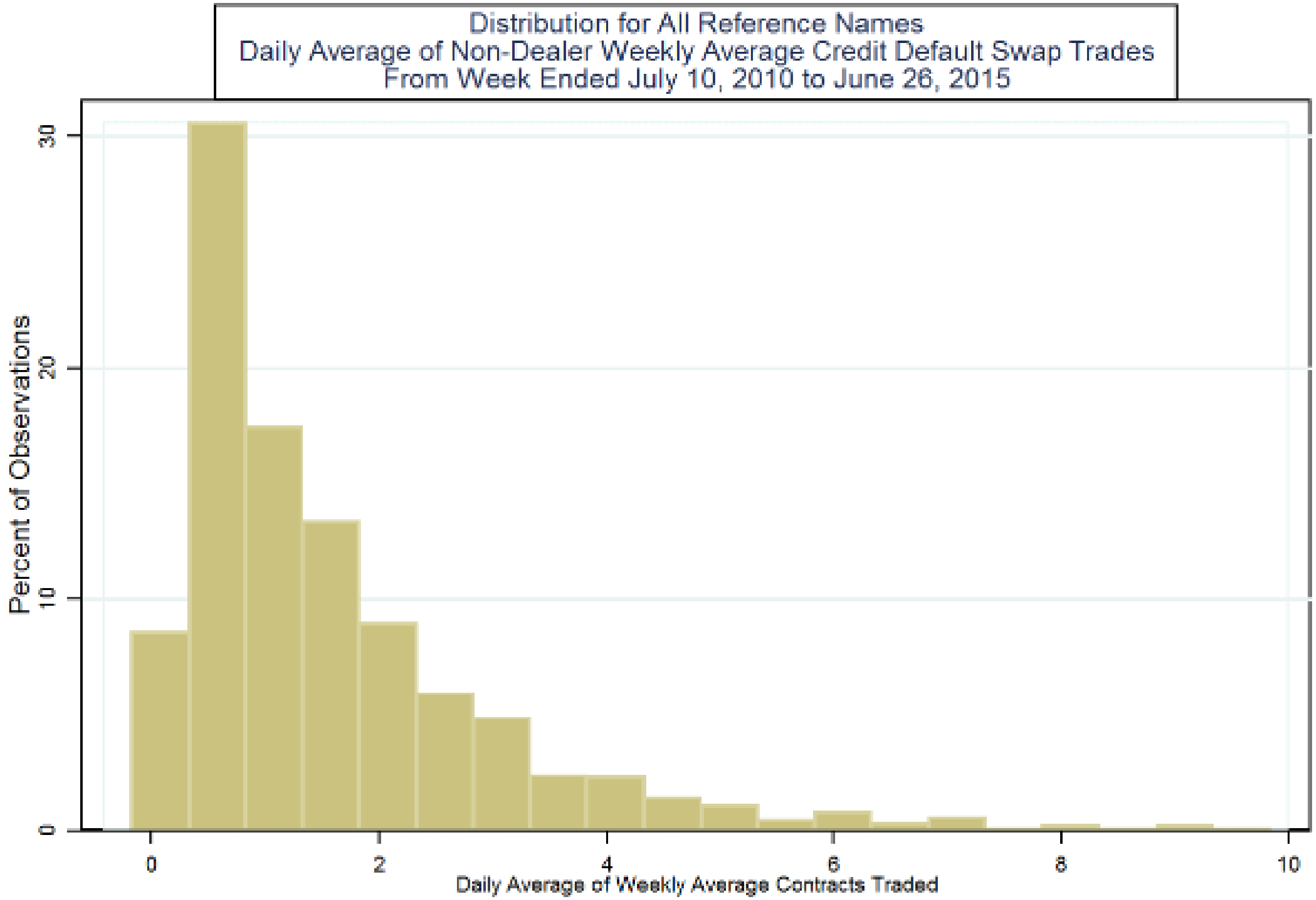
  This structure can be extended to multiple variables.

  Incorporation of correlated variables is algebraically very messy but possible.

Why Not Just Look at Credit Default Swaps?

- Fixed income investors often equate OAS spreads on actual bonds with credit default swap spreads on the same or similar instrument.
  - CDS spreads represent only the expected loss (the blue term in my equations), so we would expect that CDS spreads are persistently downward biased relative to real OAS spreads.
  - This empirical finding was reported by Bill McCoy of Factset at a Northfield seminar in Tokyo corroborating a number of reports from Kamakura.
  - Single name CDS trading volumes are extremely small so it is not clear there is fair value pricing. When you net out trades between the few big dealers, the typical volume is less *two trades per week, with the mode of the distribution being one trade per week.*

Do Single Name CDS Matter At All?



Source: Kamakura Corporation, Depository Trust & Clearing Corporation

A Bit More on CDS Spreads

- Even early evidence on CDS indicated material differences between OAS spreads and CDS spreads (Backshall, *Barra Insight*, 2003).
- Tang and Yan (2006) argue that CDS spreads include their own illiquidity premium on the order of 10 basis points, which actually reduces the *apparent difference between OAS and CDS spreads*.
- Zhang, Zhou and Zhu (2007) argue that roughly 50% of time variation in single name CDS spreads can be explained by changes in equity volatility, Cao, Fu and Zhou (2009) confirms that option implied equity volatility predicts changes in CDS spreads. Both are consistent with our Merton representation.
- Cserna and Imbierowicz (2008) finds that CDS spread are not efficient. A capital structure arbitrage strategy finds the Leland and Toft (1996) variation of the Merton model, and the structural credit model of Zhou (2001) both produce trading profits against CDS.

Basic Analytical Model of Spread Volatility

- If the OAS spread at time t is a function of four terms, the volatility of the OAS is square root of the summation of the variances and covariance of those terms.

$$sd(OAS_t)=(var(EL_t)+var(var(L_t{}^*EL_t))+var(L_t{}^*k_t)+COV())^{.5}$$

Note the higher order term in red. This will create a high degree of negative skew in distribution of returns at the individual security level. Bond investors must diversify widely to mitigate the impact of variation in expected loss, but cannot diversify the impact of variation in investor risk aversion over time.

Mitigating the Limitations

- The DTS method leaves portfolios vulnerable to inefficient risk estimation during extreme conditions or with concentrated portfolios. In addition to the "behind the scenes" credit analysis borrowed from the EE model, we are introducing an explicit process into our application software for estimating the higher moments of portfolio return distributions
- With respect to bond portfolios you can think of this as a multi-step process
  - We estimate the distribution of PD and LGD for each bond as previously described.
  - This converts to a "mixture of normal distributions" problem which provides a four parameter distribution (mean, standard deviation, skew and kurtosis) for each bond.
  - This information is converted to "normal distribution" equivalent parameters using an enhanced version of the method of Cornish and Fisher (1937). This requires knowing the sign of the holding (i.e. default risk is quite different depending on whether you are long or short).
  - For more information see http://www.northinfo.com/documents/766.pdf.

Conclusions

- While the DTS method seems intuitively sensible and has well demonstrated empirical support, we believe there are material limitations of which investors must be aware.
- To address these concerns we will introduce a number of nuances to our DTS based process that we believe will mitigate the concerns.
  - The credit risk analytics from our Everything, Everywhere risk model will influence the expected values of credit spread volatility and correlation in addition to just conditioning historical values to current spread levels.
  - Our analytical software application will incorporate "higher moment" calculations to address extent to which concentrated portfolios or high default correlation associated with crisis conditions.

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