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Dealing with Japanese and Frontier Equity Markets by Considering Non-Traditional Distributions

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Webinar February 2019

Introduction

- Global equity markets had materially negative returns during the fourth quarter of 2018. Among the largest magnitude declines was the Japanese market where the index values fell almost 20% during this period.
- It is customary in the investment industry to make certain assumptions which make asset allocation based on Modern Portfolio Theory (Markowitz, 1952, 1959) tractable. Typical assumptions include that returns are normally distributed, volatility is constant and returns are not serially correlated.
  - One clear example of non-conforming behavior would be the Japanese equity market which has been subject to powerful long term trends for nearly four decades.
- Similarly, many emerging and frontier markets have demonstrated very extreme return events (e.g. Zimbabwe, Venezuela) far more frequently than would be expected. *The key outcome is that the expected frequency of extreme events impacting global equity investors is far higher than is typically estimated.*

Baseline Assumptions

- While the usual assumptions are clearly untrue in high frequency data (see diBartolomeo, *Professional Investor*, 2007), most empirical research would be suggest that "normal and IID" assumptions would not be routinely rejected for returns when measured over longer periods such as months or calendar quarters.

- However, there are many financial markets where analysis of historical return data even at monthly periodicity would reject the usual assumptions.

- In this presentation we will illustrate how proper consideration of non-traditional return behavior (skew, kurtosis, serial correlation, correlation jumps) should impact our understanding of the risk of various national equity markets.

Background Reading

- diBartolomeo, Dan. "Fat Tails, Tall Tales and Puppy Dog Tails", *Professional Investor*, Autumn, 2007.
- diBartolomeo, Dan. "Fat Tails, Liquidity Limits and IID Assumptions", Northfield News, March 2008. http://www.northinfo.com/documents/285.pdf
- diBartolomeo, Dan. "The Volatility of Financial Assets Behaving Badly: The Example of the High Yield Bond Market", *Northfield News*, March 2013, http://www.northinfo.com/documents/546.pdf
- diBartolomeo, Dan. "Risk Modeling of Frontier Equity Markets", *Northfield News*, December 2011, http://www.northinfo.com/documents/496.pdf
- diBartolomeo, Dan and Howard Hoffman. "See the Really Big Picture: How Conflict and Corruption Impact Financial Markets. Northfield Essay, June 2015 http://www.northinfo.com/documents/496.pdf

A Classic Example: Nikkei 225

- We will start our discussion with an empirical example of daily returns to the Nikkei 225 index from May 17, 1949 to February 22, 2019.
  - Removing weekends and holidays yields a sample of 16,726 trading days.
- Let's look at basic summary statistics
  - Mean Daily Return = .031%
  - Standard Deviation = 1.21%
  - Skew = -.262
  - Excess Kurtosis = 9.08
  - AR(1) = .015
- If we run the Jarque Bera test for normality the JB value is 152.
  - For a large sample like this, anything over 6 is statistically significant
  - We can reject hypothesis that the returns are normally distributed.
  - The AR(1) value is also significant at the 98% level so we can also the hypothesis of random returns

Let's Dig a Little Deeper

- If must reject the assumptions of normal and IID, what estimate of volatility would be a better representation of the daily return behavior
- To represent the skew and kurtosis in the distribution we can apply the Cornish-Fisher expansion (1937), https://en.wikipedia.org/wiki/Cornish–Fisher_expansion to find an adjusted volatility value that takes into account the higher moments. Northfield uses a proprietary version of CF that enforces monotonic behavior.
  - The equivalent daily volatility for the Japanese equity market is 2.17, much higher than the traditional 1.21% standard deviation.
  - If we do a simple annualization of the 1.21%, it is 19.32%, while 2.17 with simple annualization would yield 34.65%
- To account for the serial correlation, the simplest correction is to multiply the volatility by the ratio (1+R)/(1-R) where R is the AR(1) coefficient.
  - This yields annualized volatility of 35.76%, which is much higher than investors would typically ascribe to the Japanese equity market.

Deeper We Go

- For the nearly 70 years of the sample, the annual geometric mean return for the Nikkei 225 is 7.15% per annum.
- The mean daily return times the number of trading days per year produces an estimated annual arithmetic return of 7.93%
- Assuming long term returns were normally distributed and serially uncorrelated, this implies an annual volatility of 12.4%, far less than our other estimates of:
  - 19.32% (simple daily annualized)
  - 34.65% (daily corrected for higher moments)
  - 35.76% (daily corrected for higher moments and AR(1)
  - Investors who have long guaranteed survival times can ignore higher moments. The difference between 12.4% and 19.2% is usually ascribed to *daily returns having something like a T-5 distribution.*

The Key Question

- So how likely was it that the Nikkei 225 would produce a fourth quarter with a negative 20% return?
  - November 1, 2018 EE model estimate 11.63% = .03%
  - Implied 70 year volatility 12.4 = .1%
  - Daily 70 year annualized = 1.93%
  - Daily 70 year corrected for higher moments = 12.4%
  - Daily 70 year corrected for higher moments and AR(1) = 13.2%

- Given that we don't see negative 20% quarters nearly half the time, what is the implied "smoothing" associated with a longer time horizon if the effect was ALL serial correlation
  - Implied AR(1) [19.2/12.4] = .21
  - Implied AR(1) [34.65/12.4] = .47
  - Implied AR(1) [35.76/12.4] = .48

Why Would We Expect Normality?

- As of 10/31/2018 the expected average annual volatility of the members of the Nikkei 225 was approximately 26.24%, while the expected volatility of the index was 11.63%.
- This is equivalent to saying that the equities in the Nikkei 225 have such high average correlation that the "effective number of independent return distributions" is (26.24/11.63)^2 = 5.08.
- This is far fewer than would usually be considered sufficient for the T distribution to converge to the normal distribution (> 35).
- So our data checks out. The Nikkei 225 daily data looks like a T-5 distribution with a modest amount of serial correlation. With this information we can calculate the probability of a loss of any given size over any desired time interval.
- Investors might ascribe the high correlations to "Keiretsu" structures or the high influence of BOJ and MITI.

Now Let's Flip to Frontier Markets

- There have been many examples of the near total collapse of numerous economies or related equity markets.
  - Russia 1917, German 1930s, China 1949
  - In the 1990s the Russian and Asian Currency crises
  - More recently Zimbabwe and current Venezuela
  - The photo below is *an actual $100 TRILLION Zimbabwe bill.* I received it as a gift, but it has a current value of around US$3 on EBay.

Risk and Frontier Markets

- One of the obvious risks in frontier markets is the small but non-zero potential for an entire market or economy to collapse.
  - This sort of event cannot be adequately modeled simply by looking at historical data because the continued existence of the market *implies that the catastrophe hasn't happened yet.*
  - To capture risk correctly we have to model two states of the future, one in which collapse takes place and another in which the catastrophic event does not take place.
- This two state world is very similar to situation of a bond either defaulting or not defaulting.
  - There is a high probability that nothing bad will happen but a small probability that something very bad, but not fully known will occur.
  - Just like Japan, the effective independence of securities in frontier markets is very low because the common geopolitical risk.

Estimating the Probability of National Collapse

- One approach to estimating the probability P of a collapse would be to explicitly estimate the likelihood of a sovereign debt default.
  - If country X is borrowing in US$ at 10% when the US Treasury is borrowing for the same term at 3%, that extra 7% is the investor's risk premium for a potential sovereign default.
  - One explicit way to calculate this probability is the model presented in
    - Belev, Emilian and Dan diBartolomeo "A Structural Model of Sovereign Credit Risk", *PRMIA New Frontiers in Risk Management Award Winner.* 2013.
    - The presentation version of this material was http://www.northinfo.com/documents/531.pdf
  - Obviously if we know the probability of collapse P, we know the probability of the collapse not happening (1-P). It can be modeled as a mixture of two normal distributions.

A More Qualitative View

- Geopolitical instability is likely to be evidenced by numerous features of economic statistics and national structure.
  - Countries with high degrees of legal and business corruption. This is often evidenced by a low ratio of equity market capitalization / GDP as per Schleifer and Vishny (1993).
  - Countries with low GDP per capita
    - Poor hungry people get upset and will undertake a literal regime change.
  - Countries with legal systems that were inherited from colonial occupation and that are counter to local religion and customs.

- Numerous economic studies such as Barro (2005) and Gabaix (2009) suggest that disaster risk is more important to investors than traditional risk measures such as beta or volatility.

Mixture Distributions

- The moments of a mixture of normal distributions are:

$$\mu = \sum_{i=1}^{n} p_i \mu_i$$

$$\sigma^2 = \sum_{i=1}^{n} p_i(\sigma_i^2 + \mu_i^2) - \mu^2$$

$$skew = \frac{1}{\sigma^3} \sum_{i=1}^{n} p_i(\mu_i - \mu)[3\sigma_i^2 + (\mu_i - \mu)^2]$$

$$kurtosis = \frac{1}{\sigma^4} \sum_{i=1}^{n} p_i[3\sigma_i^4 + 6(\mu_i - \mu)^2\sigma_i^2 + (\mu_i - \mu)^4]$$

Cornish Fisher Expansion (Again, Yes I Know)

- Using the method of Cornish and Fisher (1937) we can approximate the quantiles of a distribution given the moments of that distribution. The rather lengthy algebraic details are given at:
  - https://en.wikipedia.org/wiki/Cornish–Fisher expansion
- Widely used in evaluating option risk for known delta/gamma.
- We've added a couple of our own "bells and whistles" this method including some monotonicity requirements.

My Usual Example of a Catastrophe

- Assume a frontier equity market with an observed volatility of 17% annually but with a 2% chance of a 90% loss in an economic collapse.

- The big "left tail" in the return distribution creates very large negative skew and positive kurtosis.
  - Skew = -3.59
  - Kurtosis = 18.91

- Using the CF method, the equivalent volatility goes up from 17% to 33.1% annually.
  - *Traditional volatility modeling methods dramatically understate risk.*

- Being short this market has an equivalent volatility of 5.65% (a collapse is a profit opportunity if you are short).

Conclusions

- While portfolio theory has always assumed that equity return distributions are normal and IID, this is clearly untrue for higher frequency data such as daily observations.
- The correlation across equity securities in certain markets (e.g. Japan) is so high that a market with literally thousands of securities will effectively not have enough independent distributions to meet the requirements of the Central Limit Theorem.
- As such no expectation of normality is appropriate in many instances. Empirical analysis of the entire history of the Nikkei 225 is consistent with our view of the operative mechanism at work.
- The common geopolitical risk in frontier markets represents another example, one where we can explicitly estimate some probability of economic collapse.
- We can better model these events inclusive of higher moments via mixture distribution and the Cornish Fisher transformation.