Can Credit Risk be hedged with Equity Options?

Davide Avino[a] Enrique Salvador[b]

[a] University of Liverpool

[b] Jaume I University

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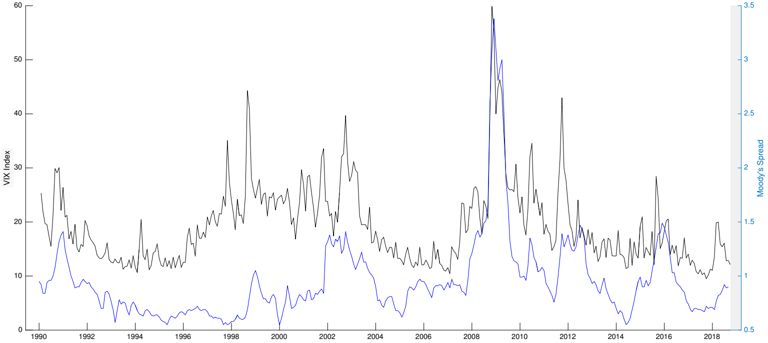
Credit and Equity Option Markets



Related Literature

- Theoretical link between credit instruments and equity options
  - Carr and Wu (2010, 2011)
- Hedging credit default swaps (CDS) with put options
  - JPMorgan (2006)
- Implied volatility as an important determinant of credit spreads
  - Collin-Dufresne et al. (2001), Cremers et al. (2008), Cao et al. (2010)
- **Contributions**:
  1. Derivation of hedge ratios based on option pricing theory
  2. Empirical tests of the validity of the theoretical hedge ratios
  3. Study the interrelations between credit, equity and equity option markets

Defining Hedge Ratios

- We derive theoretical hedge ratios of credit spread (CS) to put option value (P):

$$hr_P = \frac{\partial CS}{\partial P} P \tag{1}$$

Merton (1974)

- According to Merton (1974), the corporate bond yield spread of maturity τ is given by:

$$CS(\tau) = -\frac{1}{\tau}\ln\left(\Phi[h_2(d,\sigma_V^2\tau)] + \frac{1}{d}\Phi[h_1(d,\sigma_V^2\tau)]\right) \tag{2}$$

- And the equity value of a firm is a European call option on the asset value V:

$$E = V\Phi[h_1(d,\sigma_V^2\tau)] - De^{-r\tau}\Phi[h_2(d,\sigma_V^2\tau)] \tag{3}$$

where

$$d = \frac{De^{-r\tau}}{V}$$

$$h_1(d,\sigma_V^2\tau) = \frac{-(\sigma_V^2\tau/2 - \ln(d))}{\sigma_V\sqrt{\tau}}, h_2(d,\sigma_V^2\tau) = \frac{-(\sigma_V^2\tau/2 + \ln(d))}{\sigma_V\sqrt{\tau}}$$

Geske (1979)

- Geske (1979) shows that an equity option can be regarded as a compound option on the firm's asset value. For the case of a put option of maturity τ_1 with strike price K, P would be equal to:

$$\begin{aligned} P &= De^{-r\tau}\Theta[-h_3(\bar{d},\sigma_V^2\tau_1), h_2(d,\sigma_V^2\tau); -\sqrt{\tau_1/\tau}] \\ &\quad -V\Theta[-(h_3(\bar{d},\sigma_V^2\tau_1)+\sigma_V\sqrt{\tau_1}), h_1(d,\sigma_V^2\tau); -\sqrt{\tau_1/\tau}] \\ &\quad +Ke^{-r\tau_1}\Phi[-h_3(\bar{d},\sigma_V^2\tau_1)] \end{aligned} \quad (4)$$

where

$$\bar{d} = \frac{\bar{V}e^{-r\tau_1}}{V}, h_3(\bar{d},\sigma_V^2\tau_1) = \frac{-(\sigma_V^2\tau_1/2 + \ln(\bar{d}))}{\sigma_V\sqrt{\tau_1}}$$

and $\bar{V}$ is the solution to the following equation:

$$V\Phi[h_2(d,\sigma_V^2\tau)+\sigma_V\sqrt{\tau-\tau_1}] - De^{-r(\tau-\tau_1)}\Phi[h_2(d,\sigma_V^2\tau)] - K = 0$$

Theoretical Hedge Ratios

- We can then combine Merton (1974) and Geske (1979) to derive theoretical hedge ratios of credit spread (CS) to put option value (P) as:

$$hr_P = \frac{\partial CS}{\partial P}P = \frac{\partial CS}{\partial V}\frac{\partial V}{\partial P}P = \frac{\partial CS}{\partial V}\frac{\partial V}{\partial E}\frac{\partial E}{\partial P}P \quad (5)$$

$$hr_P = \frac{\partial CS}{\partial P}P = \frac{-\frac{1}{\tau}\frac{\frac{\phi[h_2(d,\sigma_V^2\tau)]}{V\sigma_V\sqrt{\tau}} + \frac{1}{De^{-r\tau}}(\Phi[h_1(d,\sigma_V^2\tau)] - \frac{\phi[h_1(d,\sigma_V^2\tau)]}{\sigma_V\sqrt{\tau}})}{\Phi[h_2(d,\sigma_V^2\tau)] + \frac{1}{d}\Phi[h_1(d,\sigma_V^2\tau)]} - \Phi[h_2(d,\sigma_V^2\tau)]}{\Phi[h_1(d,\sigma_V^2\tau)]\Theta[-h_3(\bar{d},\sigma_V^2\tau_1), h_2(d,\sigma_V^2\tau); -\sqrt{\tau_1/\tau}]}P \quad (6)$$

Datasets

- Equity put option data from OptionMetrics
  - Focus on options which are, on average, OTM with 1-month maturity
  - Standard filters used
- 5-year maturity CDS spreads on senior unsecured debt are from Bloomberg
- Equity market data (stock prices and outstanding number of shares) from CRSP
- Company debt data are from Compustat
- Final sample of 106 firms during August 2001 - June 2014

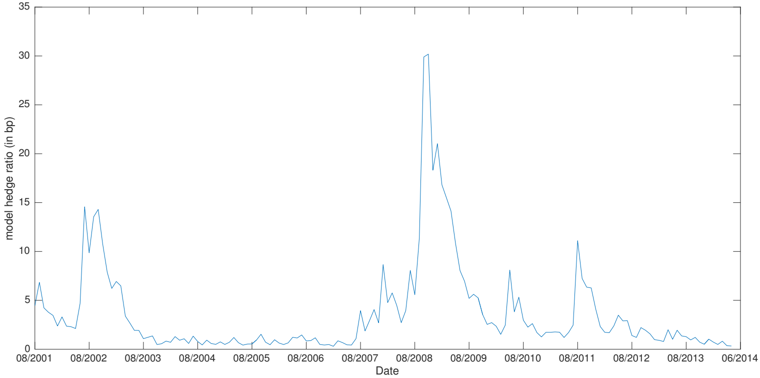
Summary Statistics for Put Options

	All	AAA-AA	A	BBB	BB
No. firms	106	13	39	43	11
Mean maturity	28.41	28.39	28.33	28.45	28.50
Median maturity	26.04	26.00	26.00	26.09	26.00
Mean moneyness	0.93	0.94	0.93	0.93	0.92
Median moneyness	0.93	0.94	0.93	0.93	0.93
Mean open interest	4,330	7,709	5,065	2,786	3,763
Median open interest	2,433	4,748	2,908	1,458	1,823
Mean delta	-0.25	-0.25	-0.23	-0.26	-0.29
Median delta	-0.21	-0.21	-0.19	-0.22	-0.26

Summary Statistics on Leverage and Volatility

	All	AAA-AA	A	BBB	BB
Panel A: Leverage					
Mean	0.36	0.48	0.39	0.28	0.38
Standard Deviation	0.08	0.08	0.06	0.08	0.10
5% Quantile	0.25	0.36	0.31	0.18	0.22
95% Quantile	0.49	0.59	0.51	0.44	0.55
Panel B: σ^E_{IMP}					
Mean	0.37	0.30	0.35	0.38	0.48
Standard Deviation	0.17	0.15	0.16	0.17	0.21
5% Quantile	0.20	0.15	0.19	0.21	0.28
95% Quantile	0.73	0.60	0.69	0.74	0.97
Panel C: σ^A_{IMP}					
Mean	0.28	0.20	0.25	0.30	0.35
Standard Deviation	0.10	0.08	0.10	0.11	0.12
5% Quantile	0.17	0.12	0.15	0.18	0.22
95% Quantile	0.49	0.36	0.45	0.53	0.63
Nobs	13,379	1,723	5,048	5,365	1,243

Time Series of Theoretical Hedge Ratios



	Mean	Std	5% Qtl	95% Qtl	Nobs
All	3.64	7.21	0.00	17.71	13,379
AAA-AA	1.77	3.13	0.00	7.06	1,723
A	2.66	5.77	0.00	12.40	5,048
BBB	4.06	8.02	0.00	20.42	5,365
BB	8.43	10.00	0.01	28.84	1,243

CDS Spread Changes and Option Returns: Summary Statistics

	All	AAA-AA	A	BBB	BB
Panel A: CDS spread changes (in basis points)					
Mean	-0.25	-0.12	-0.09	-0.20	-1.16
Standard Deviation	32.08	13.43	27.41	35.46	57.47
Skewness	1.19	1.19	1.19	1.24	1.01
Kurtosis	15.55	18.89	15.11	15.64	12.85
5% Quantile	-40.45	-16.44	-33.17	-44.25	-79.81
95% Quantile	41.89	17.05	33.13	46.01	86.18
Panel B: Option dollar returns (in U.S. dollars)					
Mean	-0.01	-0.16	-0.03	0.03	0.08
Standard Deviation	2.07	1.50	1.93	2.39	1.97
Skewness	5.45	4.77	5.72	5.56	4.88
Kurtosis	44.61	40.41	47.32	45.27	37.41
5% Quantile	-1.17	-1.32	-1.15	-1.20	-0.98
95% Quantile	2.05	1.68	2.04	2.03	2.57
Panel C: Option arithmetic returns (in decimals)					
Mean	0.09	-0.38	0.13	0.19	0.14
Standard Deviation	4.52	2.27	3.94	5.96	3.62
Skewness	5.86	5.80	5.84	6.24	4.51
Kurtosis	47.56	46.05	46.36	53.14	31.74
5% Quantile	-1.00	-1.00	-1.00	-1.00	-1.00
95% Quantile	4.02	2.34	4.83	3.57	4.90
Nobs	13,273	1,710	5,009	5,322	1,232

Regression of CDS Changes on Option Dollar Returns

- We estimate the following time-series regression for each firm j:

$$\Delta CDS_{j,t} = \alpha_j + \beta_{j,O} ret_{option_{j,t}} + \beta_{j,r} \Delta r_t^{10} + \epsilon_{j,t}$$

	All	AAA-AA	A	BBB	BB
Intercept	0.04	-0.09	0.23	0.22	-1.58
	(0.21)	(-0.77)	(1.51)	(0.80)	(-1.69)
ret_{option}	7.27	1.63	5.59	7.88	17.49
	(7.16)	(3.18)	(4.57)	(5.18)	(3.17)
Δr^{10}	-15.61	-11.99	-10.83	-16.96	-31.53
	(-7.96)	(-5.68)	(-7.60)	(-5.83)	(-2.35)
Adj R^2	0.13	0.12	0.12	0.15	0.17
Nobs	125.22	131.54	128.44	123.77	112.00

Regression of CDS Changes on Option Percentage Returns

- We estimate the following time-series regression for each firm j:

$$\Delta CDS_{j,t} = \alpha_j + \beta_{j,O} ret_{option_{j,t}} + \beta_{j,r} \Delta r_t^{10} + \epsilon_{j,t}$$

	All	AAA-AA	A	BBB	BB
Intercept	0.79	0.07	0.71	1.28	0.03
	(2.94)	(0.59)	(2.33)	(2.27)	(0.04)
ret_{option}	4.26	1.08	3.01	5.18	8.82
	(5.90)	(5.54)	(3.89)	(3.58)	(3.58)
Δr^{10}	-14.30	-11.59	-9.87	-15.06	-30.25
	(-7.18)	(-5.98)	(-6.51)	(-4.60)	(-2.43)
Adj R^2	0.13	0.14	0.11	0.13	0.16
Nobs	125.22	131.54	128.44	123.77	112.00

Hedge Ratio Regressions Based on Option Dollar Returns

- We estimate the following time-series regression for each firm j:

$$\Delta CDS_{j,t} = \alpha_j + \alpha_{j,O} hr_{P_{s,t}}(\sigma^{A}_{IMP}) ret_{option_{j,t}} + \beta_{j,r} \Delta r^{10}_t + \epsilon_{j,t}$$

	All	AAA-AA	A	BBB	BB
Intercept	-0.66	-0.17	-0.37	-0.67	-2.21
	(-3.57)	(-1.38)	(-1.64)	(-2.76)	(-1.83)
ret_{option}	0.96	0.84	0.84	1.01	1.29
	(-0.37)	(-0.42)	(-0.81)	(0.09)	(0.75)
Δr^{10}	-18.79	-12.79	-14.42	-20.09	-36.28
	(-9.74)	(-6.22)	(-8.39)	(-6.83)	(-3.00)
Adj R^2	0.20	0.16	0.15	0.26	0.22
Nobs	125.22	131.54	128.44	123.77	112.00

Hedge Ratio Regressions Based on Percentage Returns

- We estimate the following time-series regression for each firm j:

$$\Delta CDS_{j,t} = \alpha_j + \alpha_{j,O} hr_{P_{s,t}}(\sigma^A_{IMP}) ret_{option_{j,t}} + \beta_{j,r} \Delta r^{10}_t + \epsilon_{j,t}$$

	All	AAA-AA	A	BBB	BB
Intercept	-0.13	-0.00	0.10	-0.11	-1.18
	(-0.67)	(-0.01)	(0.32)	(-0.44)	(-1.06)
ret_{option}	0.77	0.56	0.76	0.80	0.89
	(-2.29)	(-3.34)	(-1.14)	(-1.28)	(-0.50)
Δr^{10}	-16.40	-12.19	-12.92	-17.65	-28.80
	(-8.34)	(-6.20)	(-7.67)	(-5.60)	(-2.26)
Adj R^2	0.21	0.17	0.17	0.26	0.24
Nobs	125.22	131.54	128.44	123.77	112.00

Hedging Effectiveness: Root Mean Square Error (RMSE)

- We compute the mean portfolio hedging error (e_t) on each month t:

$$e_t = \frac{1}{N}\sum_{j=1}^{N}[-(CV(CDS_{j,t+1}) - CV(CDS_{j,t})) + \delta_{j,t} ret_{option_{j,t+1}}]$$

	Unhedged	**Model**		**Empirical**	
	$RMSE_u$	$RMSE_h$	$\frac{RMSE_h}{RMSE_u} - 1$	$RMSE_h$	$\frac{RMSE_h}{RMSE_u} - 1$
			Panel A: In-sample analysis		
All	68,802	49,163	-0.29	51,530	-0.25
AAA-AA	35,176	33,753	-0.04	32,615	-0.07
A	66,127	50,106	-0.24	56,464	-0.15
BBB	75,144	57,394	-0.24	62,457	-0.17
BB	121,199	101,097	-0.17	238,304	0.97
			Panel B: Out-of-sample analysis		
All	78,103	56,683	-0.31	65,314	-0.16
AAA-AA	41,688	39,147	-0.06	39,415	-0.05
A	75,631	54,814	-0.28	69,370	-0.08
BBB	82,886	60,496	-0.27	74,415	-0.10
BB	125,938	92,384	-0.27	127,381	0.01

Industry Approach to Hedging

- We follow JPMorgan (2006) to determine the number of puts to buy:

$$Puts = \frac{N \times (1 - R)}{100 \times (K - E_D)}$$

	Unhedged	**JPMorgan**	
	$RMSE_u$	$RMSE_h$	$\frac{RMSE_h}{RMSE_u} - 1$
All	68,802	110,785	0.61
AAA-AA	35,176	92,666	1.63
A	66,127	99,948	0.51
BBB	75,144	144,761	0.93
BB	121,199	281,595	1.32

Do Options Have Incremental Explanatory Power for CDS Spread Changes?

- We estimate the following time-series regression for each firm j:

$$\Delta CDS_{j,t} = \alpha_j + \alpha_{j,E} hr_{E_t}(\sigma^A_{IMP}) ret_{stock_{j,t}} + \beta_{j,r} \Delta r^{10}_t + \epsilon_{j,t}$$

where $hr_{E_t} = \frac{\partial CS}{\partial V} \frac{\partial V}{\partial E} E$

- We then estimate:

$$\epsilon_{j,t} = \alpha_j + \alpha_{j,O} hr_{P_t}(\sigma^A_{IMP}) ret_{option_{j,t}} + v_{j,t}$$

	$hr_{P_t} = 1$	$hr_{P_t} = hr_{P_t}$	$hr_{P_t} = hr_{P_t} - \frac{\partial CS}{\partial V} \frac{\partial V}{\partial E}$
Intercept	0.20	0.09	0.11
	(3.21)	(2.02)	(2.35)
$hr_P \times ret_{option}$	2.46	0.14	0.10
	(5.48)	(2.39)	(2.64)
Adj R^2	0.02	0.02	0.02

Hedging Costs

	Industry		**Model**		**Model**	
	Put hedge	*No. puts*	*Put hedge*	*No. puts*	*Stock hedge*	*No. shares*
All	88,840	1,710	16,629	272	260,439	6,257
AAA-AA	74,032	1,256	8,509	151	167,350	3,436
A	73,238	1,466	13,474	230	229,944	5,135
BBB	98,729	1,840	17,686	274	254,454	6,706
BB	130,220	3,048	34,798	727	505,887	22,044

- It's more expensive to hedge portfolios of lower-rated firms
- Theoretical hedge ratios based on Merton (1974) and Geske (1979) represent the cheapest alternative

Policy Considerations

- Shifting focus from default loss hedging to mark-to-market hedging
  - Raising awareness that financial distress can come in various forms (e.g. AIG bailout)
  - No need to make assumptions about the Loss Given Default (standard to assume 50% recovery at default) and the stock price at default
  - Substantial financial incentive

Conclusion

- We derive theoretical hedge ratios of credit spreads to equity options based on the structural models of Merton (1974) and Geske (1979)
- We empirically test the model hedge ratios on a sample of North American firms for which both CDS and equity options are available
- Our results show that these contingent-claim models generate accurate predictions of the sensitivity of CDS changes to changes in the value of put options
- Relative to the empirical sensitivities, our theoretical hedge ratios improve hedging effectiveness
  - RMSE values are 15% lower for the whole sample of firms in the out-of-sample analysis
- The option's delta ($\frac{\partial E}{\partial P} P$) implied by the compound option model explains 2% of the variations of CDS spread changes which are left unexplained by the equity market