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Illiquidity Risk of Truly Illiquid Assets

and everything else

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What is Illiquidity

- The inability to execute a trade of an asset at its “fair value” at a particular point in time due to the unavailability of a willing counterparty in the trade:
  - Fair value may be defined in various ways, but it is generally considered the arm’s length transaction in the same type of asset very closely preceding the time of the desired trade.

- Illiquidity only matters if it occurs at the investor’s specific investment horizon or after:

- Different investors have differing investment horizons
  - A bank concerned with deposit coverage will want to be liquid on a daily basis
  - A sovereign wealth fund without any identifiable scheduled outlay liabilities is concerned to be liquid over 10, 20, 50 years plus
  - A DB pension fund is somewhere in between

Fallacies about Illiquidity

- *"…Illiquidity makes an asset less risky because its observable price is not volatile for a prolonged period of time…"*
  - Lack of trading masks actual market volatility.
  - Certainly this idea suits someone whose compensation depends on being perceived as having a high Sharpe ratio.

- *"…Hence, there is no illiquidity economic cost, but illiquidity economic benefit because illiquidity makes risk disappear…"*
  - This seems to justify / be justified by the high multiples of private deals in private equity over the last 3 years
  - However, unless one sees illiquid assets as a "giant mattress" there is no economic rationale behind this statement

Approaches to Illiquidity Risk and Premiums

- Phalipou, Nowak, and Franzoni, 2009 use entry and exit deal data for private equity funds as well as the Fama-French model to isolate a "premium" for private equity illiquidity

- Some time ago Northfield presented an approach where:

  - A simulation of outcomes was run, at the end of each of which a constrained optimization was performed. The constraint is the illiquidity of a sleeve of the portfolio (i.e. the untradeable asset).

  - The results were compared with simulation based on unconstrained optimizations, capturing the difference of returns and volatility, reflecting respectively the required liquidity premium and incremental risk due to illiquidity.

The Proposed Approach

- Capture the "intensity" of trades. Compare the intensity with the size of the market.
- Infer "survival" probability of an asset not being sold at the investment horizon
- Adjust the risk estimation to reflect the "survival" probability of bearing risk for longer than anticipated
- As a baseline model, assume illiquidity and market prices are orthogonal. This makes sense in mainstream market conditions. It also establishes a convenient reference point for models that includes a correlated dimension between the market and liquidity.
- Observe how the model will change with the correlation of the market and liquidity introduced.

The Data

- **Preqin**, *private equity transaction data*:
  - quarterly acquisitions, number and dollar amount, of private deals by fund partnerships since 2006
  - a range of sectors globally - Business Services, Clean Technology, Consumer & Retail, Energy & Utilities, Food & Agriculture, Healthcare, Industrials, Information Technology, Infrastructure, Materials, Telecoms & Media

- **NewRock Capital Management**, real estate transaction data:
  - quarterly trades, number and pound amounts, of CRE transactions since 1999
  - covers the UK by region: East England, East Midlands, London City, London, London West End, North East, Northern Ireland, North West, Scotland, South East, South West, Wales, West Midlands, Yorkshire

Intensity of Trades

- If we are interested in the number of occurrences of identical events in a certain time period, we are interested in Poisson or related distribution:

$$P(k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

where k is the number of occurrences, and λ is the "intensity" parameter

- If we have a sample of observations of trades of an asset that occur within a unit period (e.g. a quarter) we can estimate a distribution with intensity parameters that best fits the observed data

- A popular way to make the data fit a distribution is Maximum-Likelihood Estimation

MLE Estimation of Trade Intensity

- Take the empirical observations derived by a user-defined windows of length T in which we exhaustively divide history and measure the number of occurrences of sales of comps in each.

- Express as Poisson probabilities and multiply to get the joint the probability

- Convert to log representation, and solve for λ

$$\ln(P) = \ln\left(\frac{\lambda^k * e^{-\lambda}}{k!}\right) = \ln(\lambda^k * e^{\lambda}) - \ln(k!) = \ln(\lambda^k) - \ln(e^{\lambda}) - \ln(k!)$$
$$= \ln(\lambda^k) - \lambda - \ln(k!) = k * \ln(\lambda) - \lambda - \ln(k!)$$

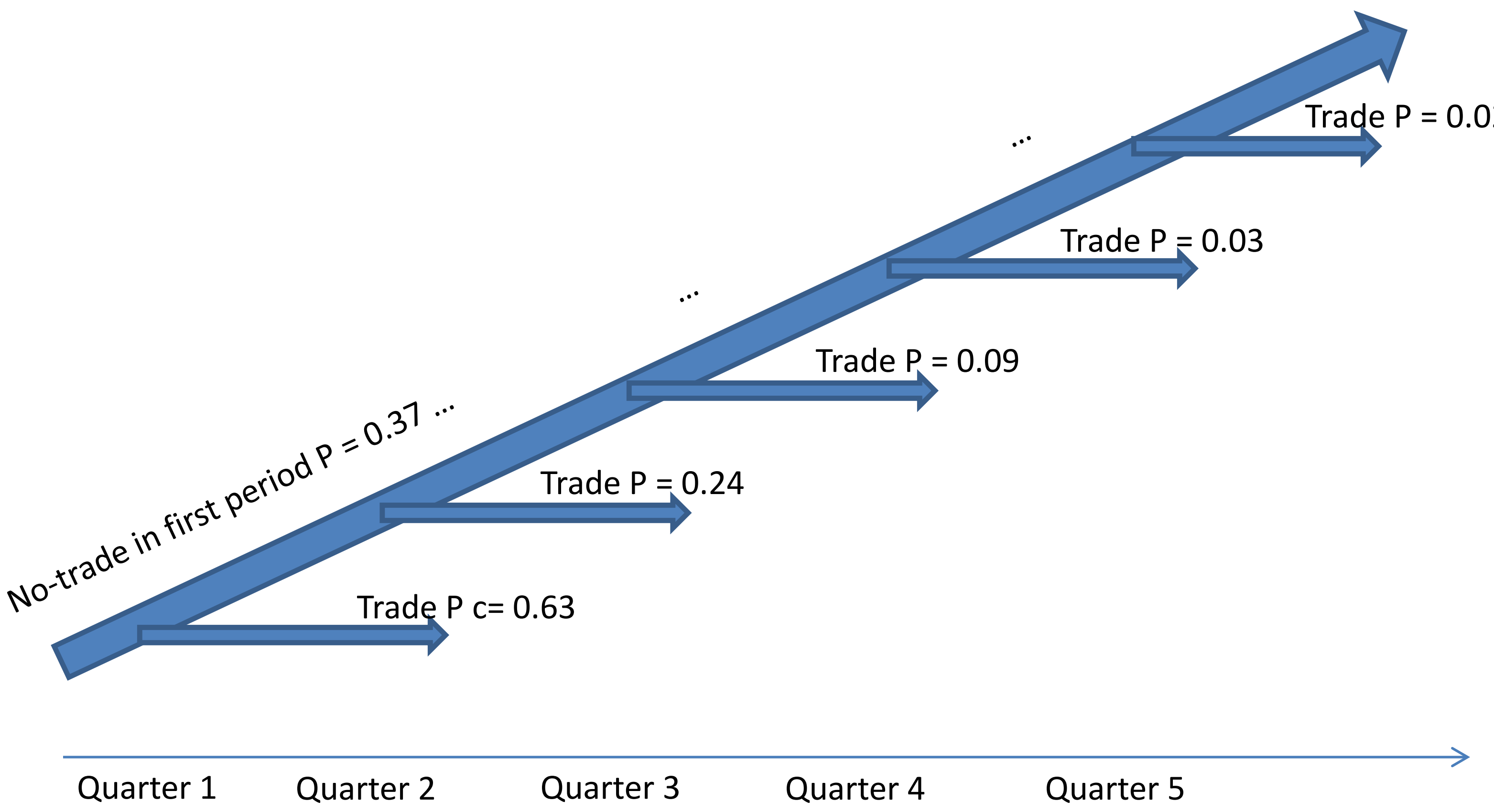
Maximize→ $\ln(P1 * P2 \ldots * PN) = \ln(\lambda) * \sum_{i=1}^{N} k_i - N\lambda - \sum_{i=1}^{N} \ln(k_i!)$

Estimated λ and P for PE and RE Markets

Office Property Location				
LONDON	**LONDON CITY**	**EAST MIDLANDS**	**SCOTLAND**	**YORKSHIRE**
Intensity of Trades				
72	**24**	**27**	**43**	**31**
Probability of trade of a single asset in a calendar quarter				
74%	**63%**	**64%**	**69%**	**66%**

Private Equity Deal Sector						
Clean Technology	**Consumer & Retail**	**Energy & Utilities**	**Healthcare**	**Industrials**	**Information Technology**	**Telecoms & Media**
Intensity of Trades						
21	**159**	**50**	**124**	**267**	**150**	**63**
Probability of transacting a single deal in a calendar quarter						
62%	**81%**	**70%**	**79%**	**84%**	**81%**	**73%**

The Tree of Trades



Adjust Volatility

- Single period volatility from market risk model:

$$\sigma^2$$

- Independent "scenario" variances combine in probability weighted fashion. Adjusted variance:

$$\sigma\_adj^2 = \sum_{i=1}^{Periods} p_i * i * \sigma^2$$

- The probability of trade $\boldsymbol{p_i}$ in the given period is calculated as:

$$p_i = \int_0^{Max\ Trades\ for\ Size\ of\ Market} \frac{x}{Size\ of\ Market} P(x, \lambda) dx$$

Adjusted Volatilities for PE and RE Assets

Office Property Location				
LONDON	**LONDON CITY**	**EAST MIDLANDS**	**SCOTLAND**	**YORKSHIRE**
Risk Model - Market Volatility				
10%	**12%**	**22%**	**11%**	**21%**
Liquidity Adjusted Volatility				
12%	**15%**	**27%**	**13%**	**26%**

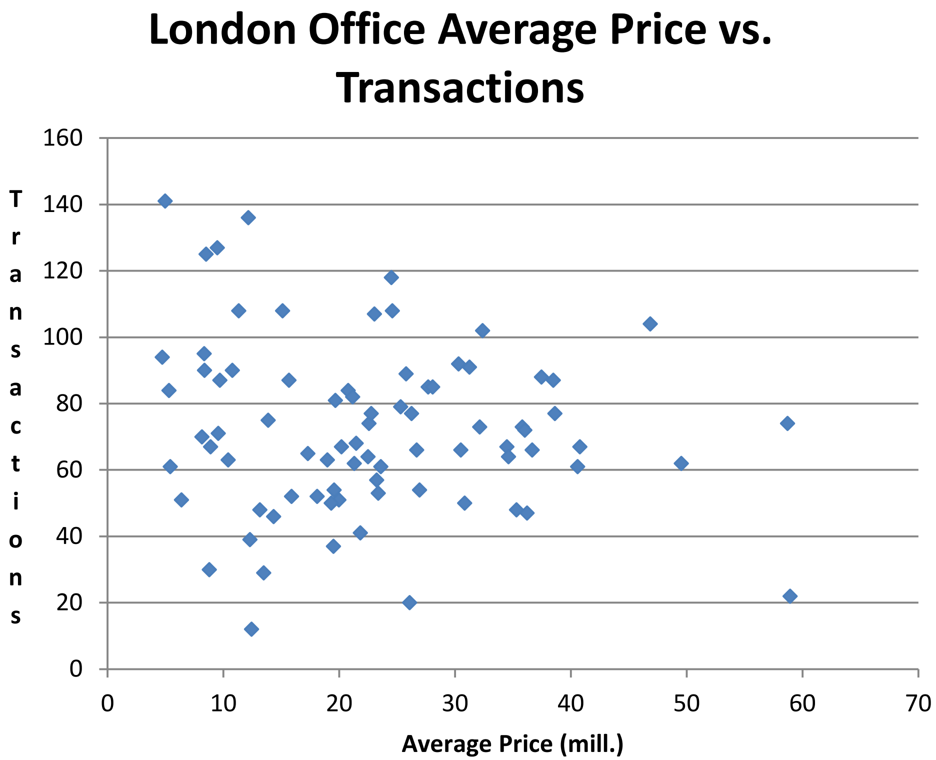
Private Equity Deal Sector						
Clean Technology	**Consumer & Retail**	**Energy & Utilities**	**Healthcare**	**Industrials**	**Information Technology**	**Telecoms & Media**
Risk Model - Market Volatility						
38%	**41%**	**20%**	**45%**	**32%**	**58%**	**32%**
Liquidity Adjusted Volatility						
48%	**46%**	**36%**	**51%**	**35%**	**65%**	**37%**

Putting in Perspective: one period holding

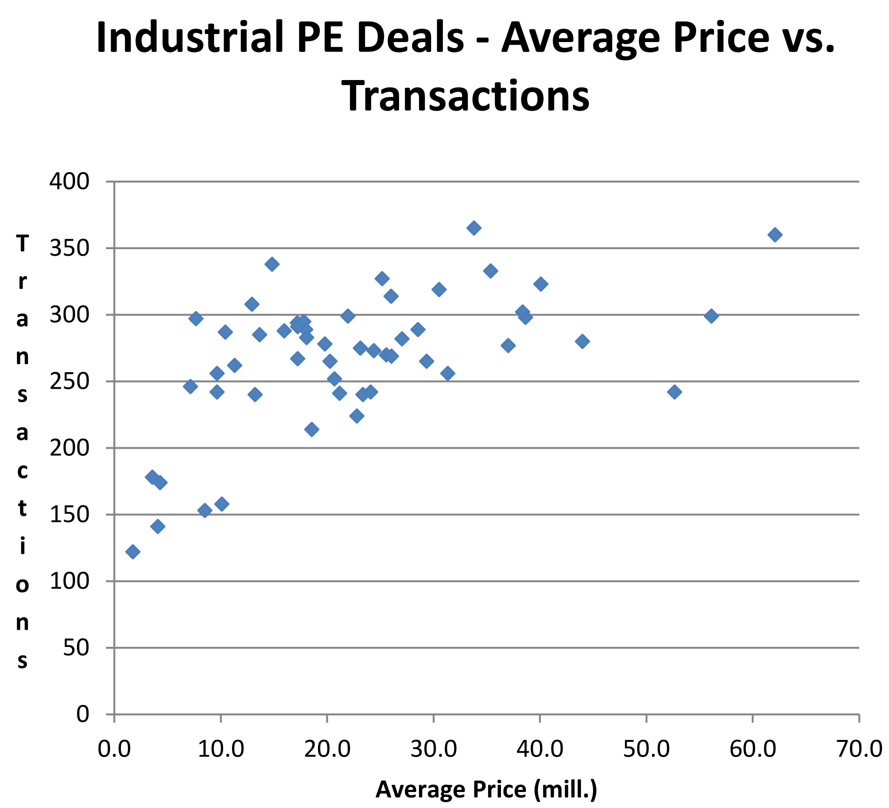
- The framework described has another realistic and intuitive economic interpretation.

- Buyers and sellers switch spots as liquidity providers and liquidity seekers at the two ends of the distribution at the end of a single period, increasing volatility as the prior result indicated.
  - When a shorter horizon owner seeks liquidity from a longer horizon investor to pass on the asset they must offer a discount to "fair value"
  - When a longer horizon investor seeks to get an investment, they must offer a premium to "fair value"
  - In between, the distance between premium and discount is split between the two parties, settling at "fair value"
  - The size of the premium-discount distance will depend on the difference of pricing of longer term investors vs. a shorter term investor

Are Asset Prices and Transactions Orthogonal



Prices and Transactions Orthogonal ? (cont'd)



Introducing Correlated Market and Illiquidity

- A Strategy:
  - Find core relationship between asset prices and number of transactions
  - A priori we don't expect it to be linear as trades should thin out both at the very top and very bottom of the "fair value" distribution
  - Compute the intensity model conditional on the average asset price
  - In the following identity the sum of the $p_i * i$ are essentially a estimable factor "k" for each value of the outcome price.

$$\sigma\_adj^2 = \sum_{i=1}^{Periods} p_i * i * \sigma^2 = k_{price\ x} * \sigma^2 = k(x) * \sigma^2$$

  - Then the distribution for the asset outcomes will follow a *higher moment*s distribution that derives from the normal density:

$$f(Return\ X) = \frac{1}{\sqrt{2\pi k(x)\sigma^2}} e^{\frac{-(x-\mu)^2}{2k(x)\sigma^2}}$$

Correlated Market and Illiquidity (cont'd)

- An Alternative Strategy:
  - Instead of computing the intensity of transactions, compute the ***intensity of moving a dollar*** in a transaction, which defines a distribution. Based on prior discussion, intuition tell us we have to do this separately for above average price markets, as well as below average price markets.
  - A correlated process of transactions and asset prices enables to back out an inferred orthogonal "residual" distribution of transactions, conditional on the asset price
  - Each such *conditional* distribution will contain information of the *conditional* probability of one asset to trade in that environment
  - This *conditional* probability will define a multiplicative adjustment to the market risk model "conventional" volatility that is *conditional* on the outcome over a single horizon.
  - As such, the resulting distribution will have fat tails and possibly a skew; a result which will be the analogous with the first strategy

The Intensity of Moving a Dollar

Putting in Perspective: Pricing Illiquidity

- If the illiquidity distribution increases volatility idiosyncratically then the illiquidity premium increases in conjunction with that volatility, if it cannot be diversified away

- If the illiquidity impact correlates with the market outcome, then it introduces higher moments, in addition to “standard deviation beta”, and all of these have to be priced in the premium for carrying that risk

- A convenient framework for the latter is Wilcox, Fabozzi (2009):

$$E[\ln(1 + L * r)] = \ln[1 + L * E(r)] - \frac{L^2 VAR}{2[1 + L * E(r)]^2} + \frac{L^3 M3}{3[1 + L * E(r)]^3} - \frac{L^4 M4}{4[1 + L * E(r)]^4}$$

- An increase in the variance and higher moments of a distribution, as derivable from our prior discussion, should be commensurate in the increase of the Expected Return E(r). The resulting increment is the price of illiquidity.

Conclusion

- We demonstrated a viable, logical, and tractable approach to liquidity risk analysis
- The discussion made it clear that illiquidity analysis only makes sense if you have a robust risk model of baseline market risk
- Northfield is at the forefront of modeling commercial real estate and private assets risk (if you don't believe us check Google). Thus, it is the only vendor which can not only reliably measure the market risk of these asset classes, but also the illiquidity risk which is an extension of the baseline market risk.
- This conceptual approach applies to any asset class. The so called "Illiquid Asset Classes" are just prominent examples of cases where other approaches to liquidity impact analysis have been scarce and deficient.

Question and Answer Session

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