Integrating real and financial investment decisions: Stock Buybacks and Their Implications

Sergei Dodzin

October 25, 2019

The Issue

- Financial conditions are favorable – low interest rates.

- Bur new fixed investment is not picking and up economists tended to overpredict investment, including investment from incentives provided by Tax Cuts and jobs act.

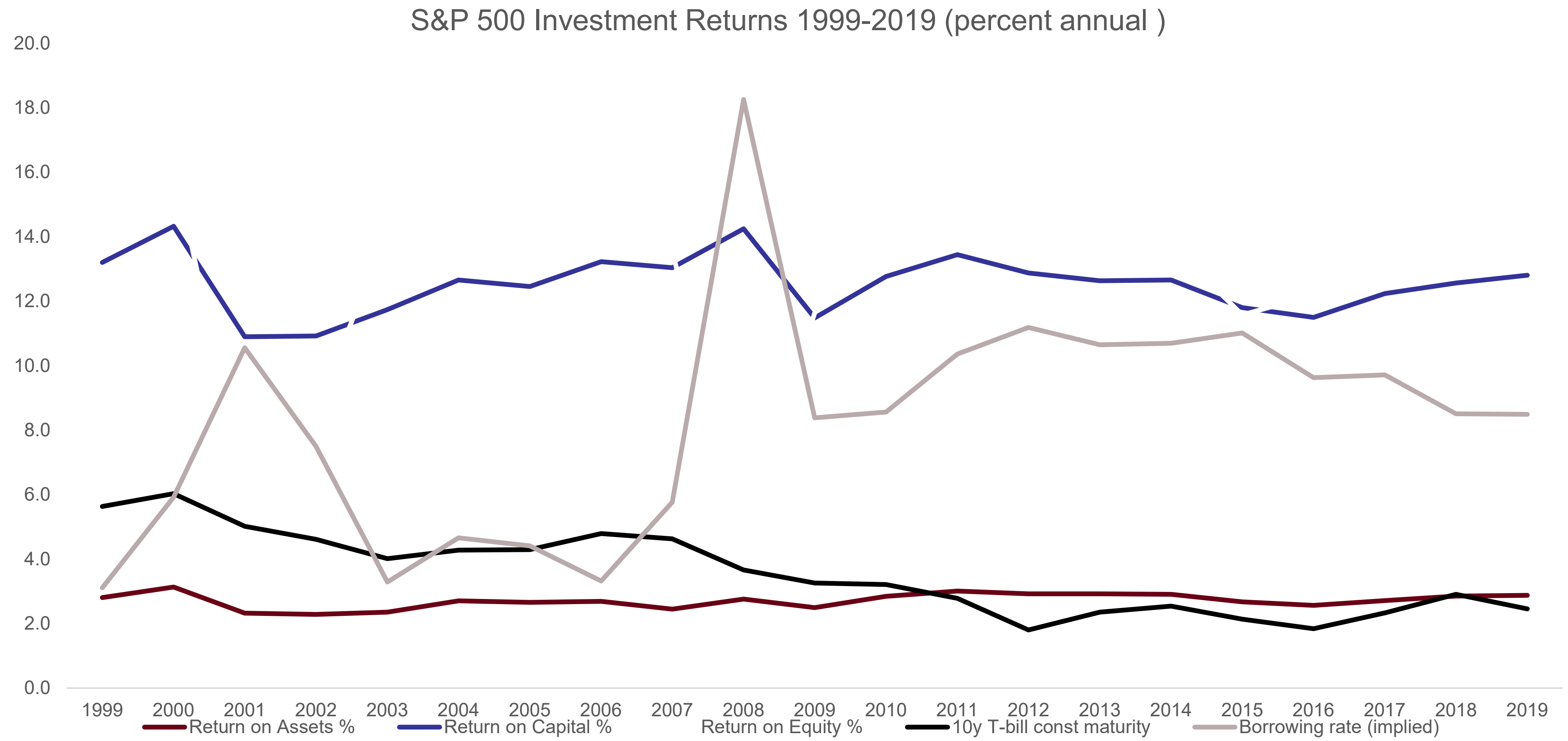
- There is a substantial share buybacks and investment in financial assets going on.

The Problem

Traditional economic models of investment and investment decisions are disconnected from the reality and objectives faced by investors in a number of ways:

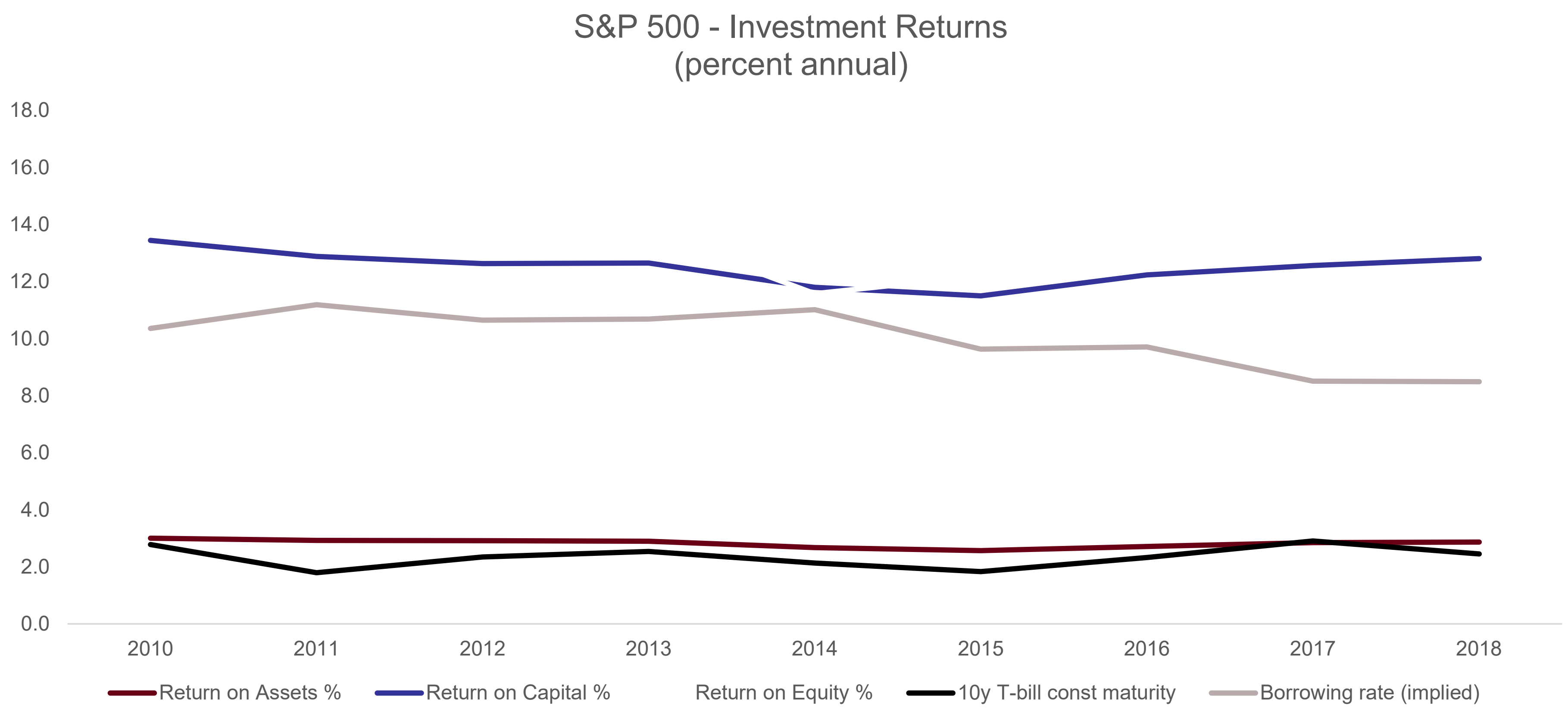
- Usually work from the supply side and assume full utilization of resources - may predict expansion which is not matched by demand
- insufficiently link real investment decisions to the firms' main objectives- such as to maximize share-holder value
- do not take into account higher irreversibility and longer horizons of real investments

• Fundamental Returns: S&P 500 (I)



Sources: S&P Capital IQ , FRED Database

• Fundamental Returns: S&P 500



Sources: S&P Capital IQ FRED Database

How to fix it?

- Financial and real investments decisions need to be considered as a joint portfolio problem.
- Leverage is not and should not be constant! Buybacks is one of the tools to adjust leverage.
- Future demand is a critical element of investment decision and need to be explained.
- The discount rates are not constant and not given! They are determined by decisions of groups of investors.
  - I will present the framework that incorporates all these features.

• More realistic framework – Linking to Investors Demand Explicitly

- Includes financial markets
- (Sufficiently) Accurately depicts critical decision making processes
- Heterogenous investors with valuing preferences
- Considers real (investment in plant etc.) decisions jointly with financial decision

• Model Setup

- Two financial assets – equity and deposit (cash)
- K investors with investor i having utility U_i
- Each investor endowed with: $N_i(t)$ number of shares and $M_i(t)$ cash
- Total number of shares- $\sum_{i=1}^{N} N_i(t)$
- Total number of cash - $\sum_{i=1}^{N} M_i(t)$

• Model Setup: Return on Equity

- Free cash flow to the firm FCF(t)
- $\mu_E(t)$ - expected return to Equity
- $1 + \mu_E(t) = \frac{E[FCF(t)]}{E(t)}$
- $\sigma_E^2(t)$ - expected variance to Equity
- $\sigma_E^2(t) = \frac{FCF(t+1)}{E(t)} = \frac{Var(FCF(t+1))}{E(t)^2}$

• Model Setup: Return per share, no buybacks

- $\mu_S(t)$ - expected return per share
- $1+\mu_s(t) = \frac{E[FCF(t+1)]}{P(t)} = \frac{E[FCF(t+1)/N(t)]}{(E(t)/N(t))} = 1+\mu_E\,(t)$
- $\sigma_S^2(t)$ - expected variance per share
- $\sigma_S^2(t) = Var\left[\frac{FCF_S(t+1)}{P(t)}\right] = \frac{Var\left(\frac{FCF(t+1)}{N(t)}\right)}{E(t)/N(t))^2} = \sigma_E^2(t)$
-

• Asset allocation problem (I)

- $w_i(t) = \theta_i \frac{1}{\sigma^2}(\mu - r)$
- $w_i(t) \ = \theta_i \frac{E(t)^2}{Var(FCF)} (\frac{\mathrm{E[FCF(t+1)}}{E(t)} - (1 + r))$
- Note that if $w_i(t)$ >1 Investor will borrow to invest in stocks.

.

• Asset allocation problem (I)

- $w_i(t) = \theta_i \frac{1}{\sigma^2}(\mu - r)$
- $w_i(t) = \theta_i \frac{E(t)^2}{Var(FCF)}(\frac{\mathrm{E[FCF(t+1)}}{E(t)} - (1 + r))$

Because the total wealth of investor is $\mathrm{P(t)}N_i(t) + M_i(t)$

$$\theta_i \frac{E(t)^2}{Var(FCF)}(\frac{\mathrm{E[FCF(t+1)}}{E(t)} - (1 + r)) \left(\mathrm{P(t)}N_i(t) + M_i(t)\right) = \mathrm{P(t)}N_i(t)$$

• Asset allocation problem

Substituting μ and σ^2 for investor i we get that

$$w_i(t) = \theta_i \frac{1}{\sigma^2}(\mu - r) = \theta_i \frac{E(t)^2}{Var(FCF)}\left(\frac{\mathrm{E[FCF(t+1)}}{E(t)} - (1 + r)\right)$$

• Individual Budget Constraint

Because the total wealth of investor is $\mathrm{P(t)}N_i(t) + M_i(t)$ we must have that

$$\theta_i \frac{E(t)^2}{Var(FCF)} \left(\frac{\mathrm{E[FCF(t+1)}}{E(t)} - (1+r)\right) \left(\mathrm{P(t)}N_i(t) + M_i(t)\right) = \mathrm{P(t)}N_i(t)$$

• Aggregate Budget Constraint

Summing up over all investors and taking into account $\sum_{i=1}^{K} \alpha_i = 1$

$$\sum_{i=1}^{K} \theta_i \frac{E(t)^2}{Var(FCF)} \left(\frac{\mathrm{E[FCF(t+1)]}}{E(t)} - (1+r)\right) \left(\alpha_i E(t) + \beta_i M(t)\right) = \left(\sum_{i=1}^{K} \alpha_i\right) E(t) = E(t)$$

And simplifying.

$$(\mathrm{E[FCF(t+1]} - (1+r)E(t)) \sum_{i=1}^{K} (\theta_i \left(\alpha_i E(t) + \beta_i M(t)\right) = Var(FCF)$$

• Equity Value

It can be shown that

$$E(t) = \frac{\mathrm{E[FCF(t+1]}}{(1+r)} - \frac{Var(FCF)}{\left(\mathrm{E[FCF(t+1]}\left(\sum_{i=1}^{k}\theta_i\alpha_i\right) + (1+r)M(t)\left(\sum_{i=1}^{k}\theta_i\beta_i\right)\right)}$$

(first degree approximation)

Where $\alpha_i = \frac{N_i}{N}\beta_i$ - allocations and θ_i and risk attitude for investor.

Risk Premium

It can also be shown that risk premium can be expressed as

$$\rho = \frac{(1+r)Var(FCF)}{\mathrm{E[FCF(t+1]}\left(\mathrm{E[FCF(t+1]}\left(\sum_{i=1}^{k}\theta_i\alpha_i\right) + (1+r)M(t)\left(\sum_{i=1}^{k}\theta_i\beta_i\right)\right) - (1+r)Var(FCF)}$$

The advantage form of risk premium explicitly shows how it can depend on individual investors' preferences and endowments.

Example

	Capital	Allocation	
Investor 1		6938	6938
		12471	
		-5534	
theta		1	
w*(inv_1)		1.80	
Shares in total		0.71	
Number of shares		571	
Investor 2		5563	5563
		5000	
		563	
theta		0.5	
w*(inv_2)		0.90	
Shares in total		0.29	
Number of shares		228.94	

Example (continued)

E[FCF]	20324
Var[FCF]	17546116.84

E[FCF]/E	1.16
mu_e	0.16
sigma_e	0.06
r	0.06
mu_e-r	0.10
(mu_e-r)/sigma_e	1.80

Buyback

Windfall	1092		
Buyback 50 shares			
Remaining		750	
Return on buyback		1.07	
E[FCF]	20324		
Var[FCF]	17546116.84		
E[FCF]/E	1.16		
E[FCF]/E (after buyback)			1.24
mu_e	0.16		
mu_e(after buyback)			0.24
sigma_e	0.06		
r	0.06		
mu_e-r	0.10		
mu_e-r after buyback			0.18

• Equity Value in the Neoclassical Framework

$$E(t) = \frac{\mathrm{E[FCF(t+1]}}{(1+r)} - \frac{Var(FCF)}{\left(\mathrm{E[FCF(t+1]}\left(\sum_{i=1}^{k} \theta_i \alpha_i\right) + (1+r)M(t)\left(\sum_{i=1}^{k} \theta_i \beta_i\right)\right)}$$

$$= \frac{\mathrm{E}[(1-T))K^{\alpha} - (1-ITC)K - \delta_k K]]}{(1+r)} - \frac{Var[((1-T))K^{\alpha} - (1-ITC)K - \delta_k K])}{\left(\mathrm{E}[(1-T))K^{\alpha} - (1-ITC)K - \delta_k K]]\left(\sum_{i=1}^{k} \theta_i \alpha_i\right) + (1+r)M(t)\left(\sum_{i=1}^{k} \theta_i \beta_i\right)\right)}$$

The firm maximizes the value of E(t):

$$E(t) -> \max$$

• Free cash flow in Neoclassical Model

Follow up work – estimates of the model, more detailed analysis of implications for the risk-free rates in the equilibrium settings.

Thank you !

• How to think about Leverage? (I)

Useful approach:

Y_t - Output of the project at time t

F_t Free cash flow

V_t Investor's own capital

w_t -fraction of the investor's own wealth invested in the free cash flow

$1 - w_t$ - fraction the investors own wealth invested in the risk free deposit (cash).

• How to think about Leverage? (II)

It is known that if

$\frac{dF_t}{F_t} \sim$ Lognormal (μ_t, σ_t) and risk free rate is r_t

then a risk neutral investor maximizing expected capital growth rate will allocate to F_T fraction

$$w_t^* = \frac{\mu_t - r_t}{\sigma_t^2}$$

(a) $w_t^* > 1$ when $\frac{\mu_t - r_t}{\sigma_t^2} > 1$ or $\mu_t > r_t + \sigma_t^2$

(b) If $r_t \rightarrow 0$ $w_t^* \rightarrow \frac{\mu_t - r_t}{\sigma_t^2}$ (zero low bound)

• How to think about Leverage? (II)

Let $F_t = (1 - T)H_t$

If $\frac{dH_t}{H_t} \sim$ Lognormal (g_t, γ_t) ,T – tax rate

$$\frac{dF_t}{F_t} \sim \text{Lognormal}\ ((1 - T)g_t, (1 - T)\gamma_t)$$

and with the risk free rate is r_t and for risk neutral investor

$$w_t^* = \frac{(1 - T)\mu_t - r_t}{(1 - T)^2 \sigma_t^2}$$

If $r_t \to 0$ $w_t^* \to \frac{1}{1-T}\frac{\mu_t}{\sigma_t^2}$

• How to think about Leverage? (II)

Example 1: Consider r_t=0.07

g_t= 0.15 γ_t= 0.3

T=0.32

Then $w_t^* = \frac{(1-0.32)*0.15-0.07}{(1-0.32)^2(0.3)^\wedge 2} = 5.53$

For the investor with a risk aversion $\theta = \frac{1}{2}$

$w_t^*(\theta) = \theta * w_t^* = \frac{1}{2} * 5.53 = 2.76$

• How to think about leverage (III)

- In general case investor i allocates to risky financial asset or combination a fraction of wealth
  - $w_i = \theta_i \frac{\mu - r}{\sigma^2}$
- Parameter θ_i can be interpreted as a degree of risk aversion:

$\theta = 1$: log (wealth growth)

$\theta = \frac{1}{2}$: mean variance

$\theta = \frac{1}{1-\gamma}$: constant relative risk aversion $u(w) = w^{\frac{1}{1-\gamma}}$

• How is this connected to share repurchases?

Example 2: Suppose in example 1 r_t goes down from 0.07 to 0.03 For the investor with a risk aversion $\theta = \frac{1}{2}$

$$w_t^*(\theta) \text{ goes up from } 2.76 \text{ to } 3.74$$

Let's the own wealth of investor is 1000 . And he borrowed 1760 to invest in the company 2760 corresponding to 2.76 above. Now he needs to increase the exposure to 3740 to get to 3.74.

One way for him is to borrow more – but there may be another way!

• How is this connected to share repurchases?

The other way may be when the company uses windfall to repurchases its shares from other investors in such a way that exposure of our investor goes to 3740 as in the example below. Therefore repurchases increase investor exposure without investor borrowing on their own!

Before repurchase

	Investor 1	Total
Exposure	2760	5000
Shares	110	200

After repurchase

	Investor 1	Total
Exposure	3740	5000
Shares	110	148

• The problems with standard neoclassical model

- More fundamental than the estimation – in reality investors follow a different decision making process!
  - The discount rates are not exogenous, they are endogenous and determined in financial markets.
  - Investors do not approach a particular firm in isolation, they consider all assets jointly of asset allocation.
  - But financial markets do not exist generally in this model, even if there are some financial features – they are usually after - thought.

• Improving application of the neoclassical model

When planning new investors the firm

a) Acts in the interest of shareholders or groups of shareholders, while being consistent with required bond returns and other necessary constraints
b) Projects future demand for product (consistently with the practice). That means that even if the classical condition $f'(K)$ calls for expansion the firm may not act on it if there is no demand.
c) Takes into account uncertainty.

• Application to Neoclassical Model (I)

$$\pi_{Kt} = a((1-T)A(K_t)^{\alpha} - (1-ITC_t)I_{1t} + \frac{1}{1+r_t}(1-T)(F(K_{t+1}) - (1-\delta)K_{t+1}))$$

- a- share of capital in production
- $K_{t+1} = K_t + (I_{1t} - \delta)K_t$
- $Y_{t+1} = (1+g_{t+1})Y_t$
- $\frac{Y_{t+1}-Y_t}{Y_t} \sim \text{Lognormal}\ (g_t, \delta_t)$

• Application to Neoclassical Model (II)

If we assume in the short run technology us fixed

$$FCF_t = (1 + r_t)[\,(1 - T)\ - (1 - \delta)a - \Big((1 - ITC)a(1 + g_t) * \\ -(1 - \delta)a\Big)] +\ ((1 - T)\Big((1 + g_t)\Big) - a_t(1 - \delta_t)^2))\,\frac{Y_t}{1+r_t}$$

By inspection if the demand is inflexible the decision by company is mostly influenced by interest rates rather than MPK – consistently with the recent observed outcomes!