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Machine Learning for Algorithmic Trading and Trade Schedule Optimization

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Abstract:
We present a machine learning technique that can be used in conjunction with multi-period trade schedule optimization in program trading. The technique is based on an artificial neural network (ANN) that determines a better starting solution for the non-linear optimization routine. Unlike many current industry approaches that use heuristics and numerical approximation, our machine learning approach solves for the exact problem and provides a dramatic improvement in calculation times. As such, we believe this this technique represents a material advance in the ability of asset managers to more efficiently manage trade execution for large transactions.

Research Topic

Previous Research and Preliminary Findings were published in:

"Machine Learning for Algorithmic Trading and Trade Schedule Optimization,"
Robert Kissell and Jungsun "Sunny" Bae
Journal of Trading, Fall 2018 Volume 13, No. 4, Pg. 138-147.

"Algorithmic Trading Methods: Applications using Advanced Statistics, Optimization, and Machine Learning Techniques," Second Edition
Robert L. Kissell, Ph.D.
Elsevier/Academic Press, 2019.

Presentation Outline

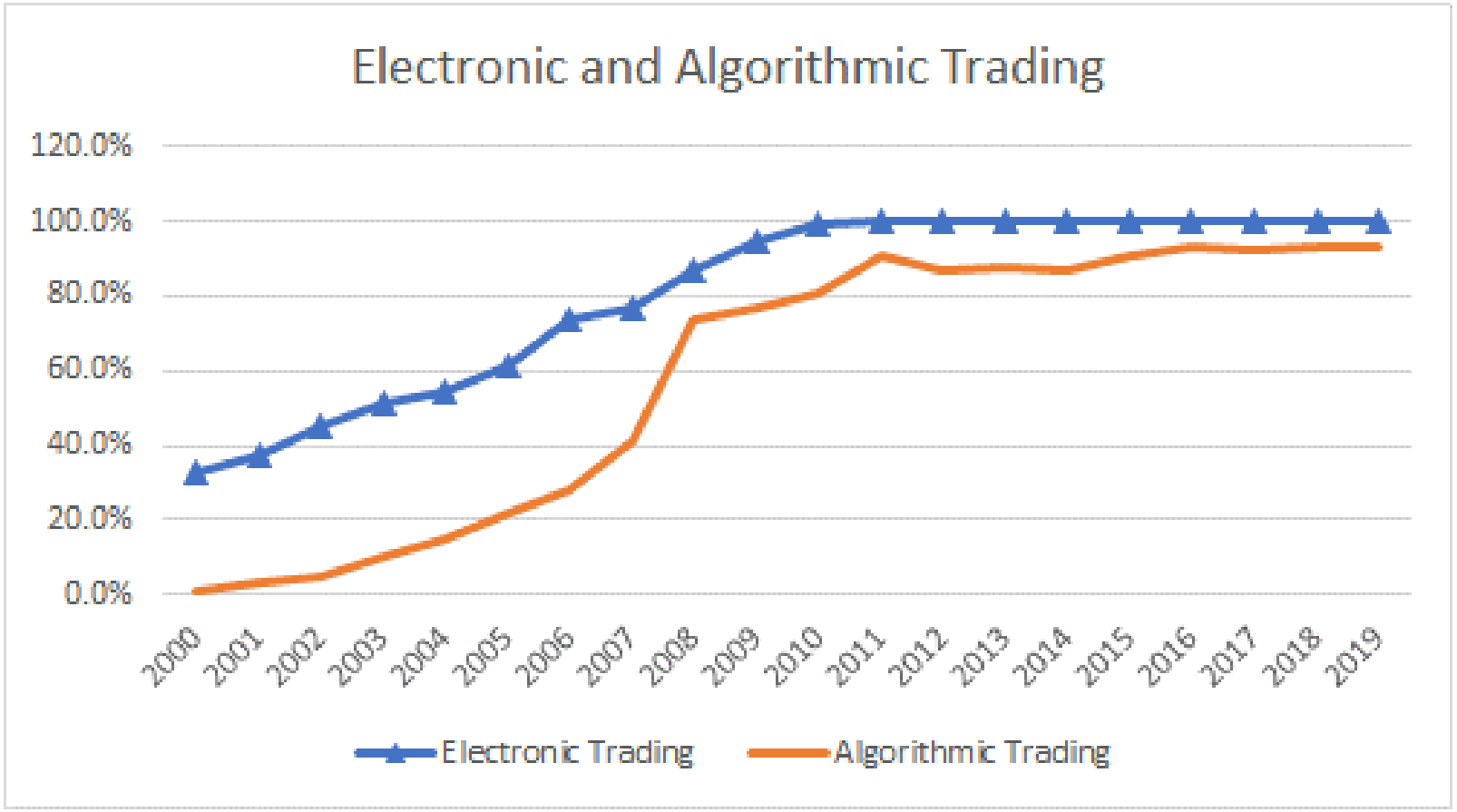
- Algorithmic Trading
- Multi-Period Trade Schedule Optimization
  - Market Impact Model, Trader's Dilemma
- Machine Learning & Data Analytics
  - Descriptive, Predictive, Prescriptive
- Convergence Algorithms
  - Newtons Method, Gradient Descent, Neural Networks
- Optimization Experiment
  - Data Inputs, Stepwise Regression, Best Variables
- Results/Conclusions

What do we want to do?

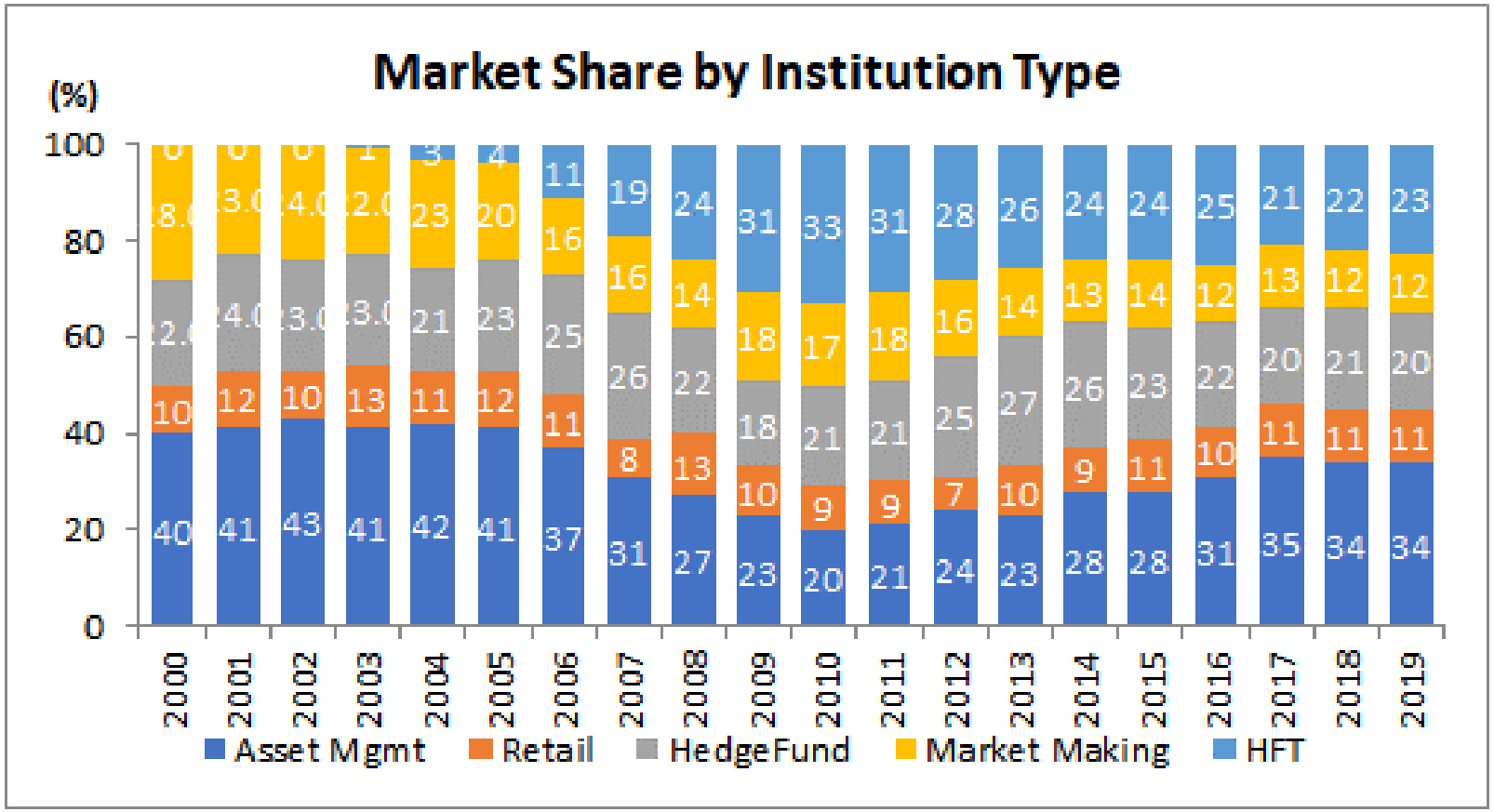
- Solve Real-Time Algorithmic Optimization Problems Faster.
- Take advantage of Profiting Opportunities when they arise, and before other market participants uncover the opportunity.
- Make Money!

Algorithmic Trading Trends

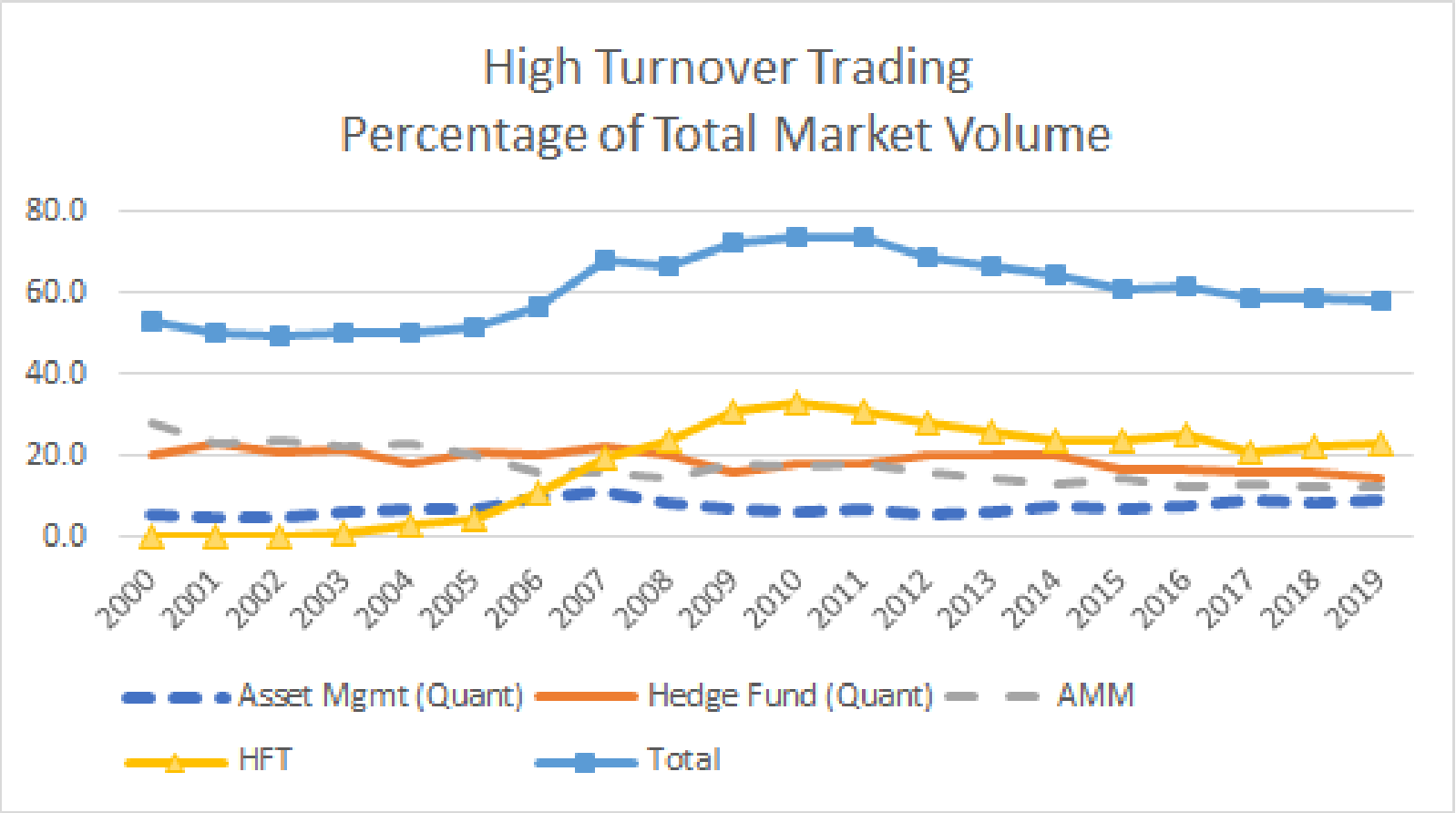
Electronic and Algorithmic Trading Volume



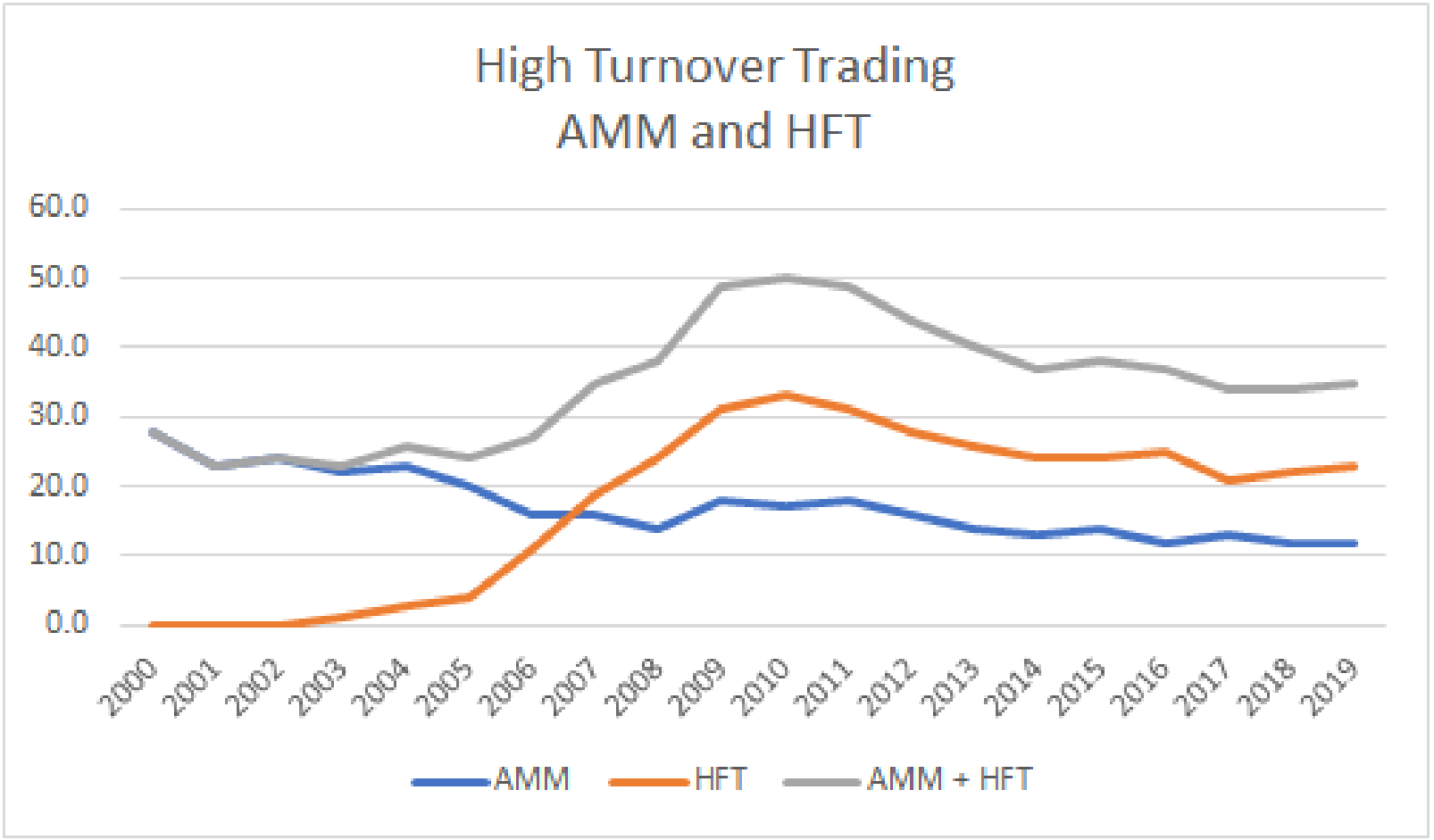
Market Share by Institution Type



High Turnover Trading



Market Share by Institution Type



Market Impact Model

I-Star Market Impact Model – POV Strategy

$$I^* = a_1 \cdot \left(\frac{X}{ADV}\right)^{a4} \cdot \sigma^{a3} \cdot P^{a5}$$

$$MI^* = b_1 \cdot I^* \cdot POV^{a4} + (1 - b_1) \cdot I^*$$

$$TR = \sigma \cdot \sqrt{\frac{1}{3} \cdot \frac{1}{250} \cdot \frac{X}{ADV} \cdot \frac{(1 - POV)}{POV}} \cdot 10^{-4}$$

Variables:

X = Shares to Trade

ADV = Average Daily Volume

σ = annualized volatility (expressed as a decimal)

P = Price

POV = percentage of volume (expressed as a decimal)

a_1, a_2, a_3, a_4, b_1 = model parameters, $a_k > 0$; $0 \leq b_1 \leq 1$

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Source: Science of Algorithmic Trading (Kissell, 2013)

Trader's Dilemma

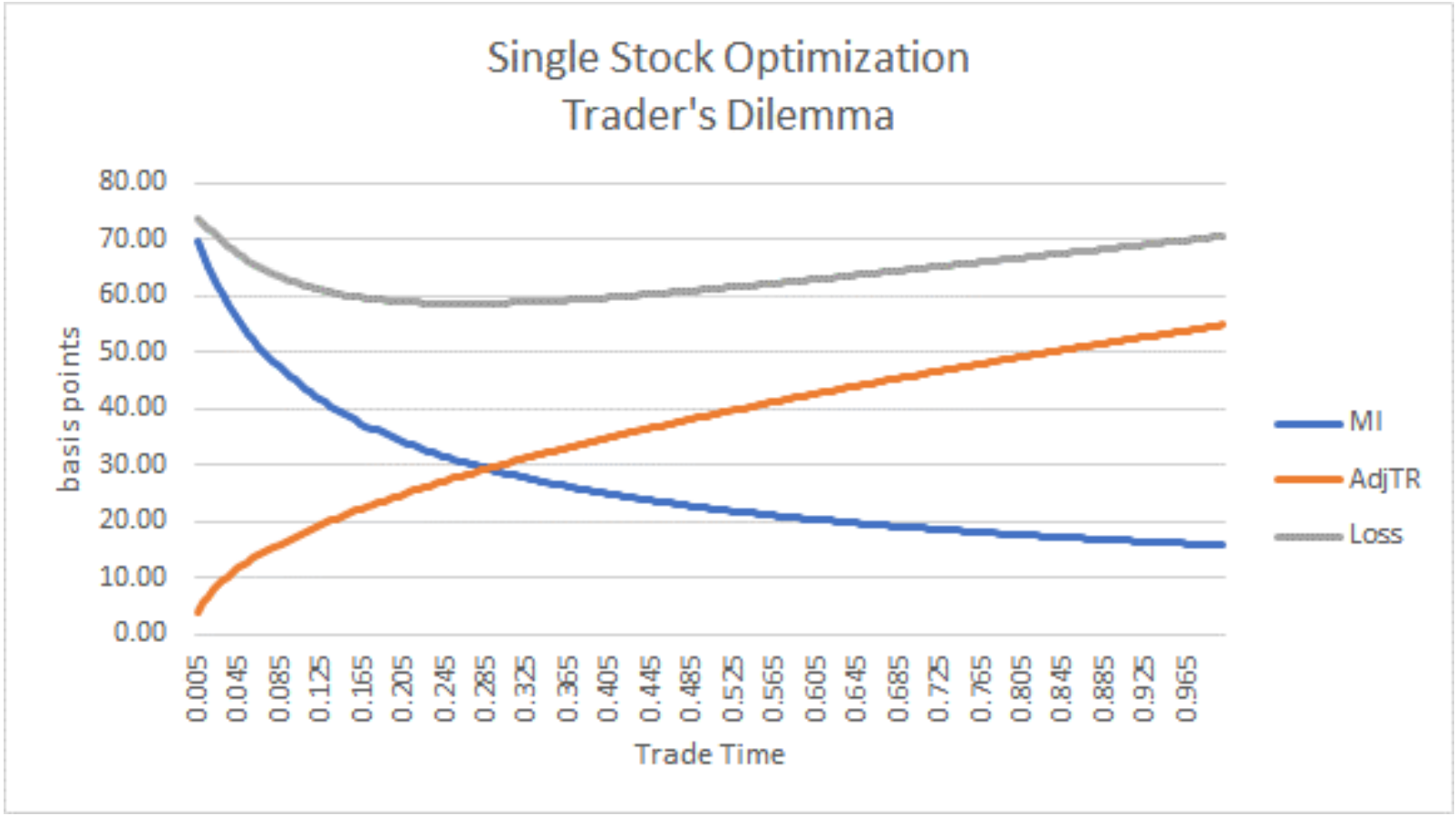
$$Min \quad MI + \lambda \cdot TR$$

Trader's Dilemma:

- Trading fast incurs high market impact cost. Trading slow exposes the trade to timing risk due to price volatility and volume uncertainty.
- Traders want to balance the trade-off between market impact cost and timing risk.

Source: Science of Algorithmic Trading (Kissell, 2013)

Single Stock Optimization: Trader's Dilemma



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Source: Science of Algorithmic Trading (Kissell, 2013)

I-Star Market Impact Model – Trade Schedule

$$\underset{POV}{Min} \; L(\$) = \sum \left((b_1 \cdot I_i^* \cdot POV^{a4} + (1 - b_1) \cdot I_i^*) \cdot X_i \cdot P_i \right)$$

$$+\lambda \cdot \sqrt{trace(R'CR)}$$

where,

$$R = f(POV)$$

s.t.

i) $\min POV \leq POV \leq maxPOV$

ii) $Cash\ Balancing$

Variables:

I^* = Instantaneous Impact

X = Shares to Trade

R= Matrix of Residuals

C= Covariance Matrix, One-Sided, ($/Share)^2

POV=Percentage of Volume (Trade Rate)

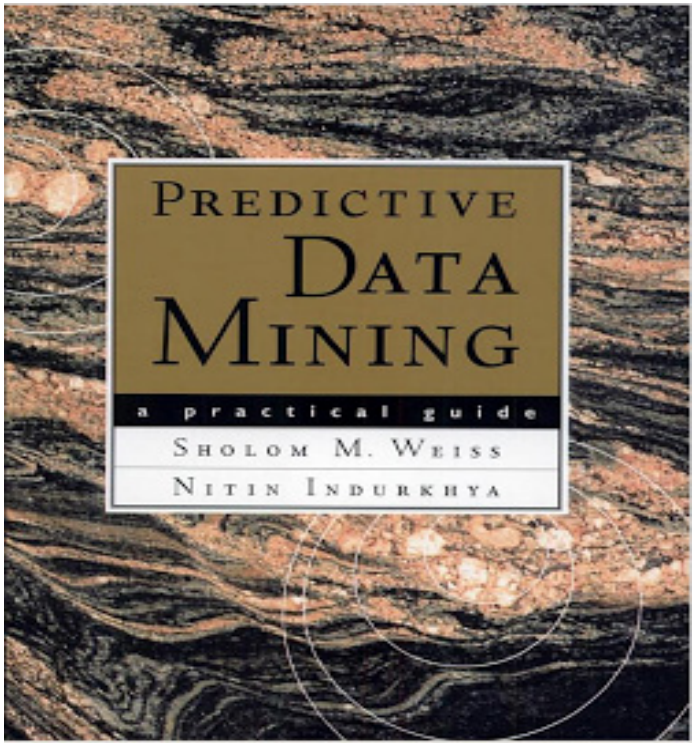
Source: Science of Algorithmic Trading (Kissell, 2013)

Machine Learning

Machine Learning

- Machine Learning gives computers the ability to learn without being explicitly programmed
  - (Arthur Samuel, 1959).
- Machine Learning Algorithms:
  - Neural Networks, Genetic Algorithms, Cluster Analysis, Decision Trees, Classification, Pattern Recognition, K-Means, Newton's Method, Gradient Descent

Predictive Data Mining (1998)



Machine Learning

- Cluster Analysis
- Classification
- Regression

Machine Learning

- Unsupervised
- Supervised

Machine Learning

- Descriptive
- Predictive
- Prescriptive

Machine Learning

- Descriptive
- Predictive
- Prescriptive

- “Semi-Prescriptive”

Fun Fact #1

- When did Sports Analytics Begin?
  - Bill James, Sabermetrics
  - Billy Bean, Oakland A's
  - Michael Lewis, Money Ball

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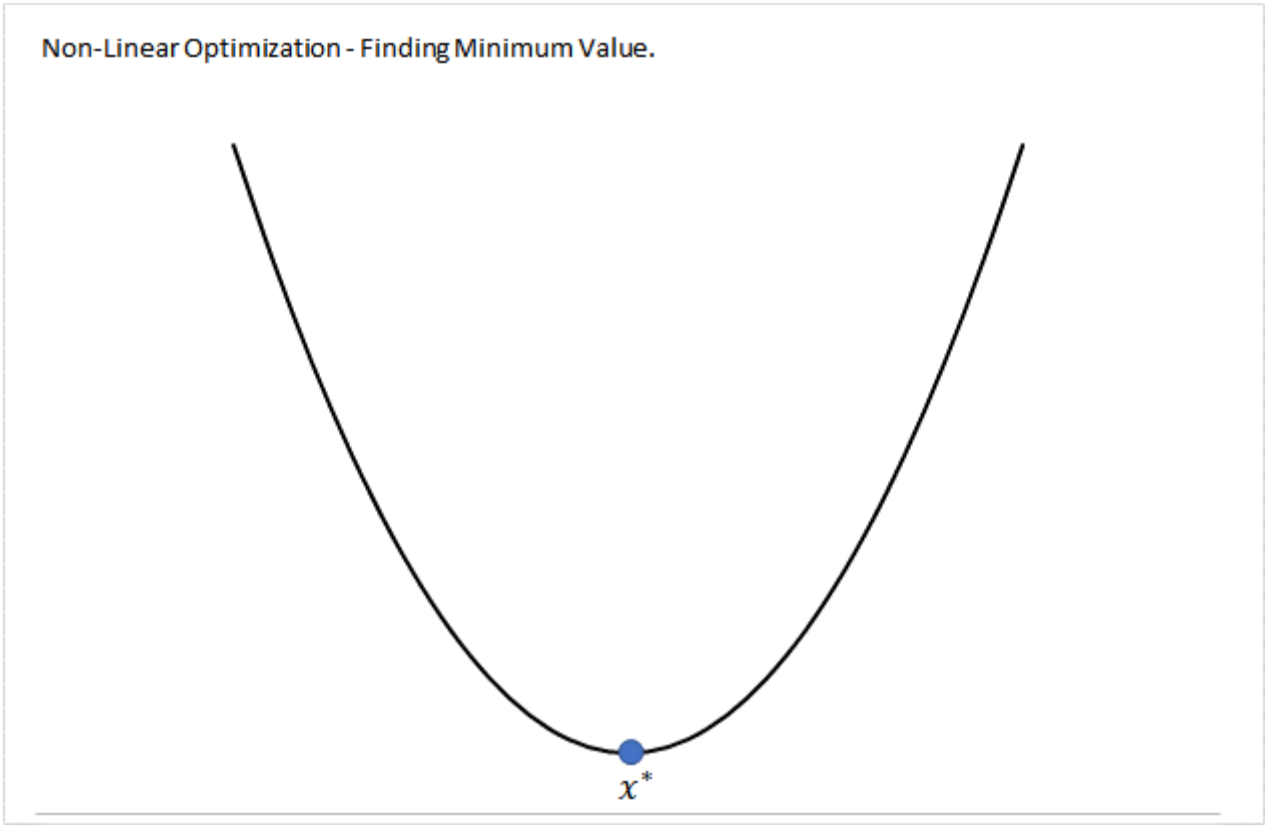
  - Billy Bean, Oakland A's
  - Michael Lewis, Money Ball

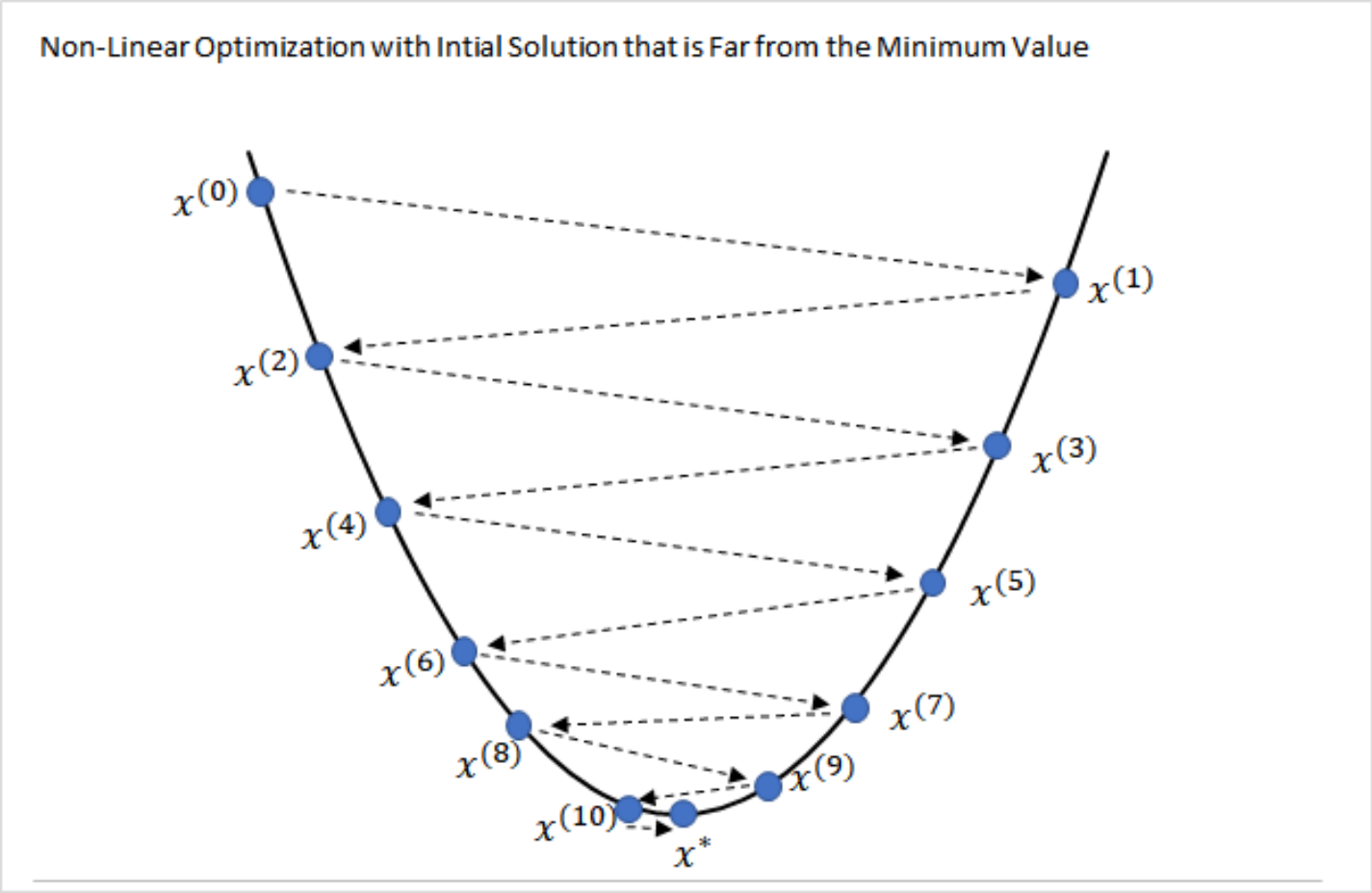
- Answer: Branch Rickey, 1950's
  - GM/Owner, Brooklyn Dodgers
  - Hired Allan Roth, Mathematician
  - Team of Mathematicians working out of a secret lab somewhere in Brooklyn.

Machine Learning Algorithms (Refresher)

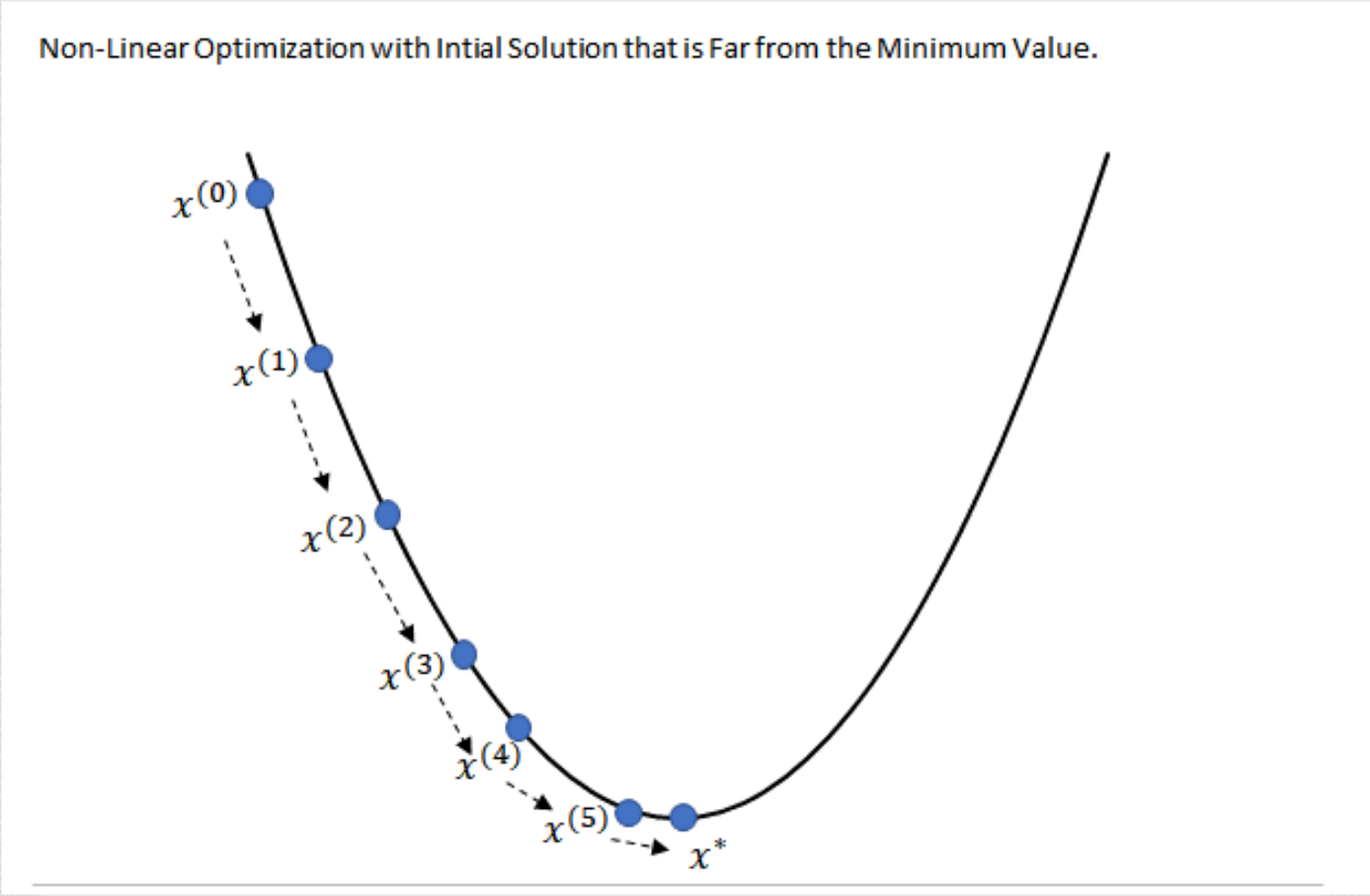
Machine Learning Algorithms



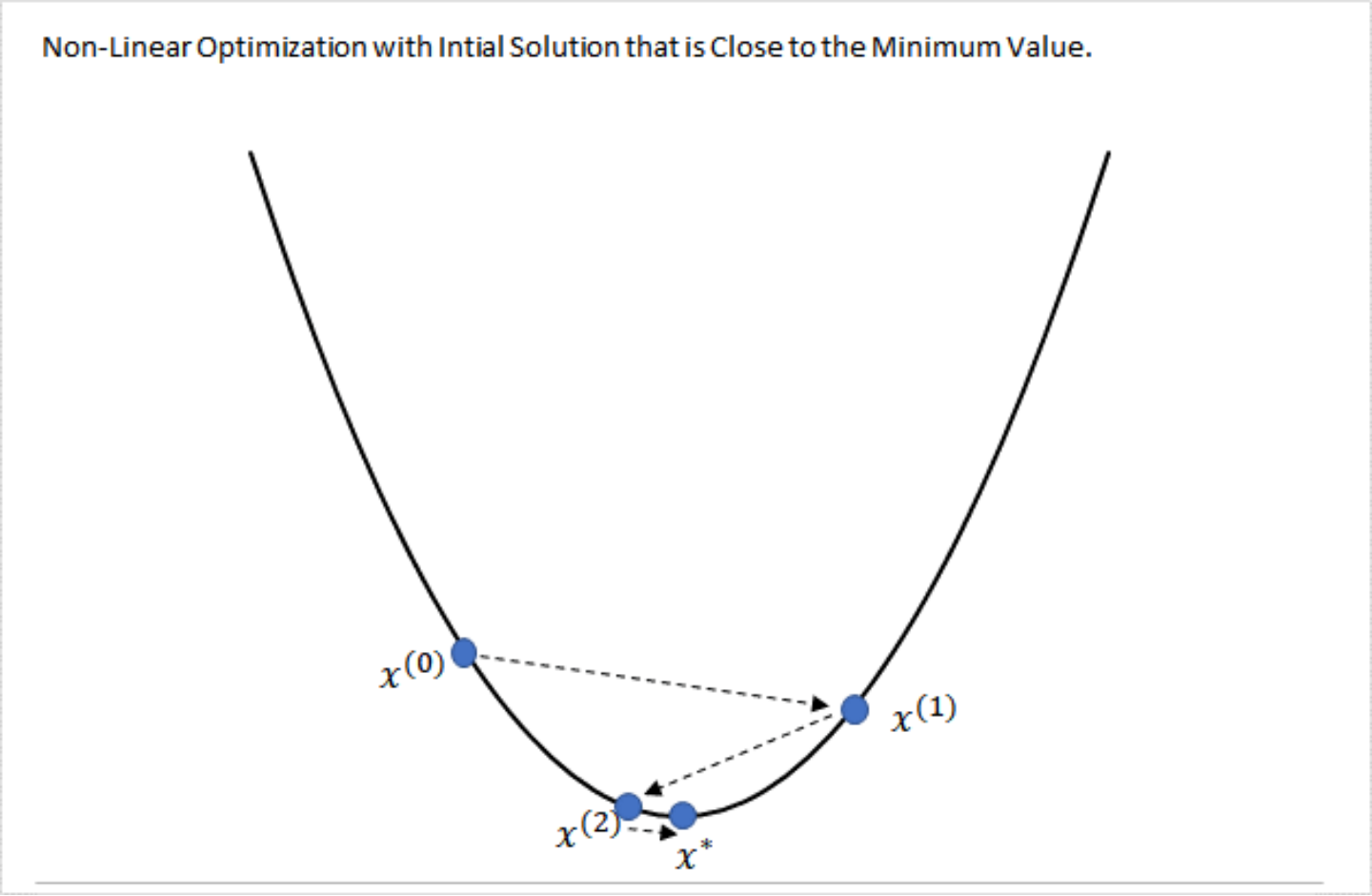
Machine Learning Algorithms



Machine Learning Algorithms



Machine Learning Algorithms



Machine Learning Algorithms

- Newton's Method
- Steepest Gradient Descent

Newton's Method

Find the zeros of a function:

$$f(x) = 0$$

Updating Function:

$$x^{i+1} = x^i - \frac{f(x^i)}{f'(x^i)}$$

Solve for x:

$$x = \sqrt[3]{73}$$

Rewriting:

$$x^3 - 73 = 0$$

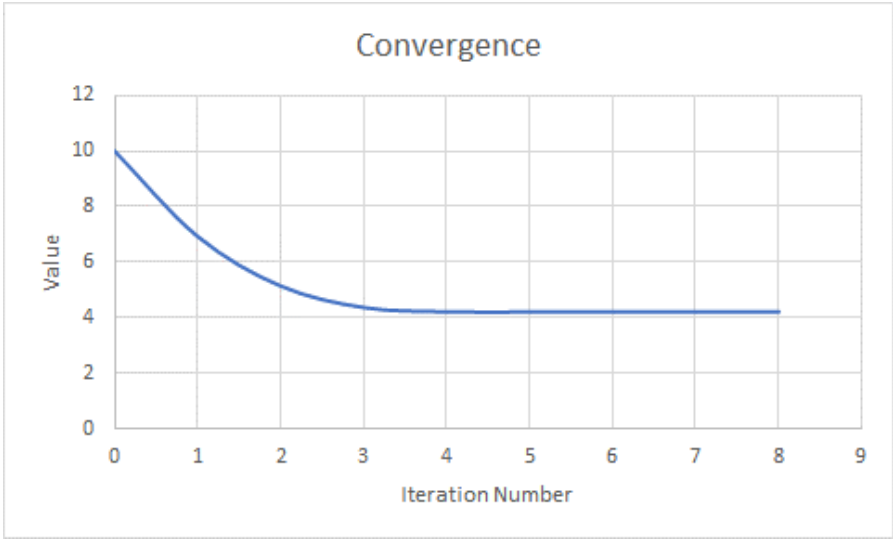
Functions:

$$f(x) = x^3 - 73$$

$$f'(x) = 3x^2$$

Machine Learning: Newton's Method (Cubed Root of 73)

Iteration	x	f(x)	f'(x)	Chg
0	10	927	300	
1	6.91	256.93937	143.24430	-3.09
2	5.11628555	60.92582	78.52913	-1.7937145
3	4.34044834	8.77184	56.51848	-0.7758372
4	4.18524527	0.30992	52.54883	-0.1552031
5	4.17934753	0.00044	52.40084	-0.0058977
6	4.17933920	0.00000	52.40063	-8.331E-06
7	4.17933920	0.00000	52.40063	-1.661E-11
8	4.17933920	0.00000	52.40063	0



Gradient Descent Method

Simple Linear Regression

$$\hat{y}_i = b_0 + b_1 x_i$$

Loss Function:

$$L = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = \frac{1}{n} \cdot \sum_{i=1}^{n} (y_i - b_0 - b_1 x_i)^2$$

Partial Derivatives:

$$\frac{\partial L}{\partial b_0} = \frac{1}{n} \cdot \sum_{i=1}^{n} -2 \cdot (y_i - \hat{y}_i) = \frac{-2}{n} \cdot \sum_{i=1}^{n} (y_i - b_0 - b_1 x_i)$$

$$\frac{\partial L}{\partial b_1} = \frac{1}{n} \cdot \sum_{i=1}^{n} -2 \cdot (y_i - \hat{y}_i) \cdot x_i = \frac{-2}{n} \cdot \sum_{i=1}^{n} (y_i - b_0 - b_1 x_i) \cdot x_i$$

Machine Learning Update Function:

$$b_k^{i+1} = b_k^i - \alpha \cdot \frac{1}{n} \cdot \frac{\partial L}{\partial b_k}$$

$\alpha = learning\ rate$

Update Function #1:

$$b_0^{i+1} = b_0^i - \alpha \cdot \frac{1}{n} \sum_{i=1}^{n} -2 \cdot (y_i - \hat{y}_i)$$

$$b_1^{i+1} = b_1^i - \alpha \cdot \frac{1}{n} \sum_{i=1}^{n} -2 \cdot (y_i - \hat{y}_i) \cdot x_i$$

Update Function #2:

$$b_0^{i+1} = b_0^i + \alpha \cdot \frac{2}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)$$

$$b_1^{i+1} = b_1^i + \alpha \cdot \frac{2}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i) \cdot x_i$$

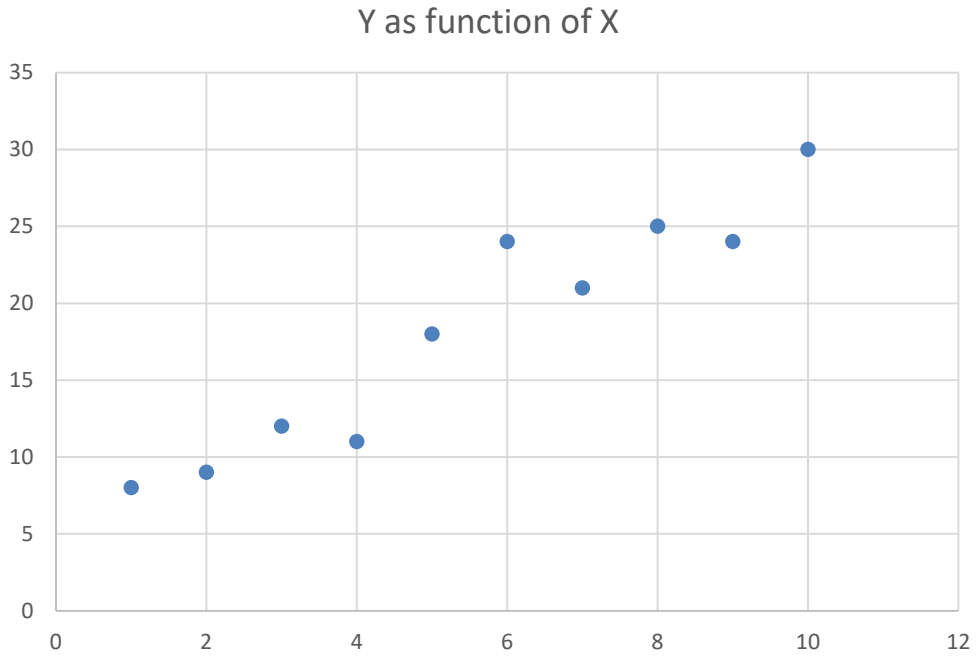
Update Function #3:

$$b_0^{i+1} = b_0^i + \alpha \cdot \frac{2}{n} \sum_{i=1}^{n} (y_i - b_0 - b_1 x_i)$$

$$b_1^{i+1} = b_1^i + \alpha \cdot \frac{2}{n} \sum_{i=1}^{n} (y_i - b_0 - b_1 x_i) \cdot x_i$$

Gradient Descent Method: Example

Linear Regression Data		
obs	Y	X
1	8	1
2	9	2
3	12	3
4	11	4
5	18	5
6	24	6
7	21	7
8	25	8
9	24	9
10	30	10



Update Function:

$$b_0^{i+1} = b_0^i + 0.01 \cdot \frac{2}{10}\sum_{i=1}^{10}\left(y_i - b_0^i - b_1^i x_i\right)$$

$$b_1^{i+1} = b_1^i + 0.01 \cdot \frac{2}{10}\sum_{i=1}^{10}\left(y_i - b_0^i - b_1^i x_i\right) \cdot x_i$$

Starting Solution:

$b_0 = 2.5$

$b_1 = 3.0$

Learning Rate:

$\alpha = 0.01$

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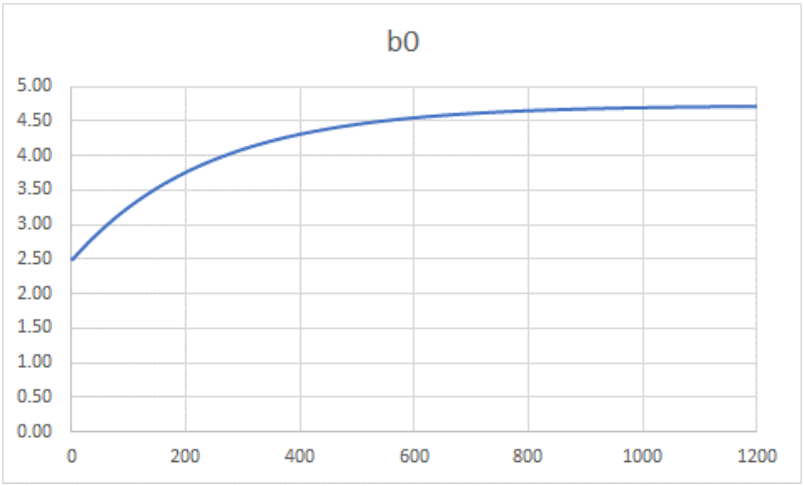
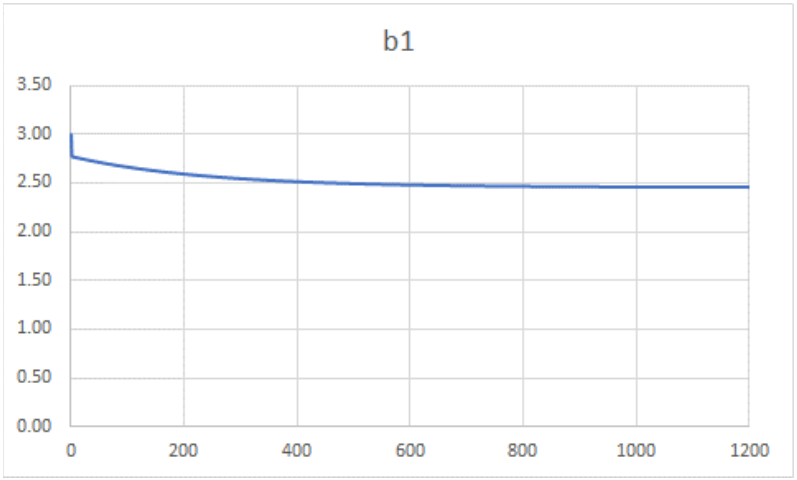
Gradient Descent Method: Example



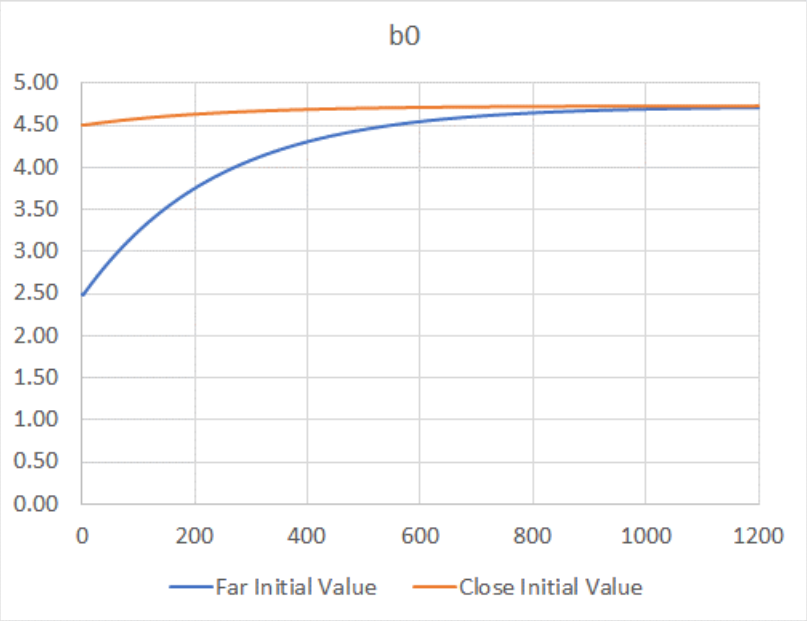


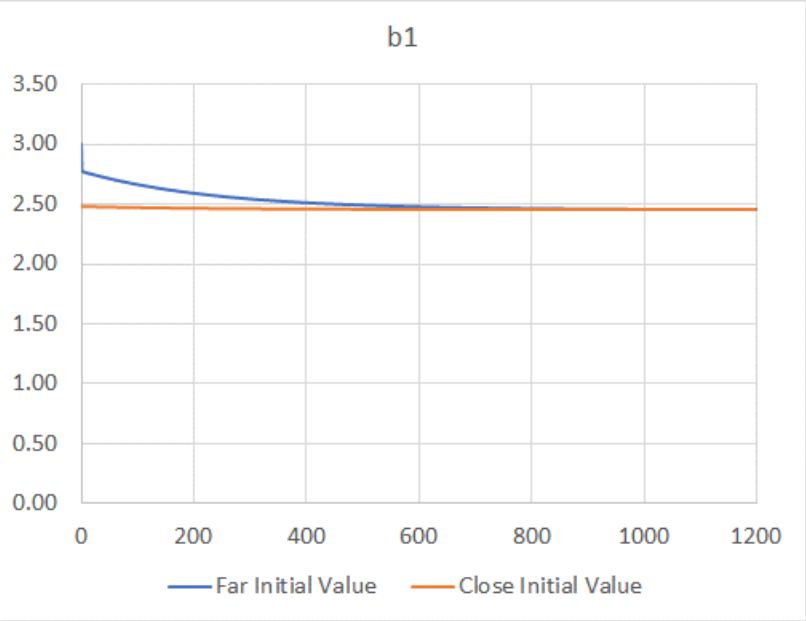
Iterative Result Using Gradient Descent Method

Iteration	b0	b1		Iteration	b0	b1
0	2.5000000	3.0000000		1190	4.7182513	2.4506512
1	2.4880100	2.7815900		1191	4.7183147	2.4506421
2	2.4962749	2.7720846		1192	4.7183777	2.4506331
3	2.5054201	2.7689892		1193	4.7184406	2.4506241
4	2.5147229	2.7672713		1194	4.7185031	2.4506151
5	2.5240286	2.7658529		1195	4.7185654	2.4506061
6	2.5333042	2.7645030		1196	4.7186274	2.4505972
7	2.5425428	2.7631722		1197	4.7186892	2.4505883
8	2.5517430	2.7618499		1198	4.7187507	2.4505795
9	2.5609046	2.7605338		1199	4.7188119	2.4505707
10	2.5700278	2.7592233		1200	4.7188729	2.4505620

	b0	b1
Initial Value	2.50	3.00

Gradient Descent Method: Example

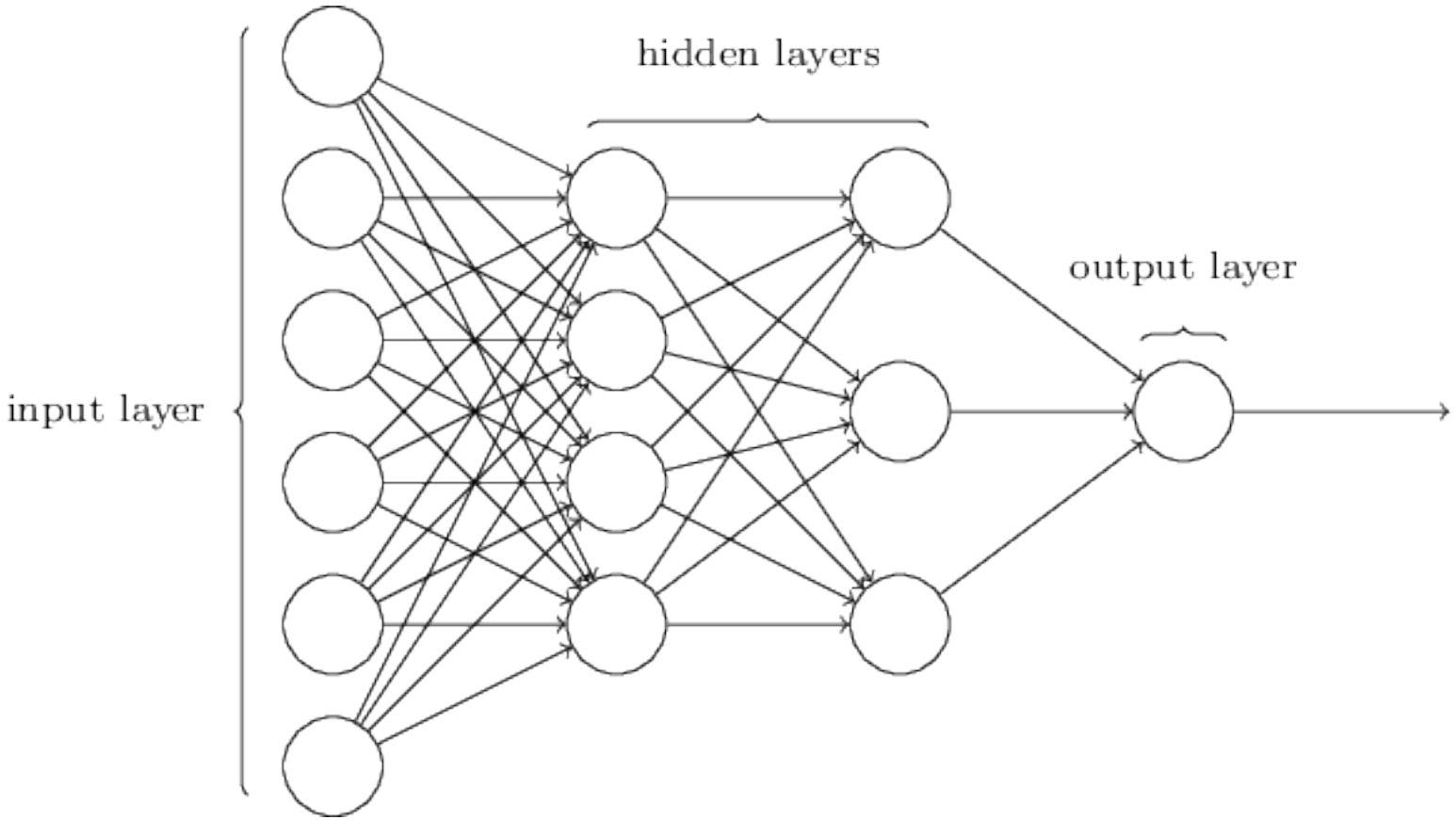




	b0	b1
Far Initial Value	2.50	3.00
Close Initial Value	4.50	2.50

Neural Networks

Neural Network



Experiment:

Multi-Period Trade Schedule Optimization using Machine Learning

Question:

Question:

- Can Machine Learning (Neural Networks) be used to find a better initial starting solution for Non-Linear Optimization?

Question:

Question:

- Can Machine Learning (Neural Networks) be used to find a better initial starting solution for Non-Linear Optimization?

Answer:

- Yes We Can!

Experiment:

Data Generating Process:

- 21 Basket Sizes ($n = 10, 25, 50, 75, \ldots, 450, 475, 500$).
- 100 Simulated Trades for each.
- 2,100 different baskets - 21 different basket sizes and 100 optimizations.
- Total of 526,000 "x-input" samples.
  - Each of the 21 baskets had a different number of stocks.

- Orders Sizes: 0.01% ADV to 50% ADV.
- One-Sided and Two-Sided Baskets
  - Two-Sided: 10% Buy and 90% Sell,…., 90% Buy and 10% Sell

- Compiled statistics for (18) different "x-variables" for each stock.

- Solved Multi-Period Trade Schedule Optimizer
  - Solved for Optimal POV Rate
  - This Optimal POV Rate is then used in the NNET Training

Experiment:

Stepwise Regression Approach:

- Step 1: select one of the fifteen variables to include in the NNET training
- Step 2: determine the best NNET structure, e.g., determine number of layers and nodes that provide the best results for the input variables.
- Step 3: record the error term, e.g., the standard deviation of the difference between the actual y and the estimated y from the neural network.
- Step 4: repeat for each input variable.
- Step 5: determine which variable was the best to include based on the lowest error.

Experiment:

Finding Important Variables

- Once we determine the best input variable that input variable was always included as one of the input factors.
    - Repeated Step 1 – Step 5 to find the next most important variable.
    - Added one variable at a time.
- In total, we trained 120 different NNET to find the best set of input variables.
- For each training, we had to determine the NNET structure that provided the lowest error term.
- On average, we trained 5 NNET structures for each data set.
    - Resulted in 500 different NNET training scenarios.
- Our machine learning experiment, was quite time consuming.

Neural Network Data:

Input Data

- Order Size
- Volatility
- Side = 1, -1
- Lambda
- “a4” Model Parameter
- IStar
- Temp MI
- Perm MI
- Timing Risk (VWAP)

Input Data

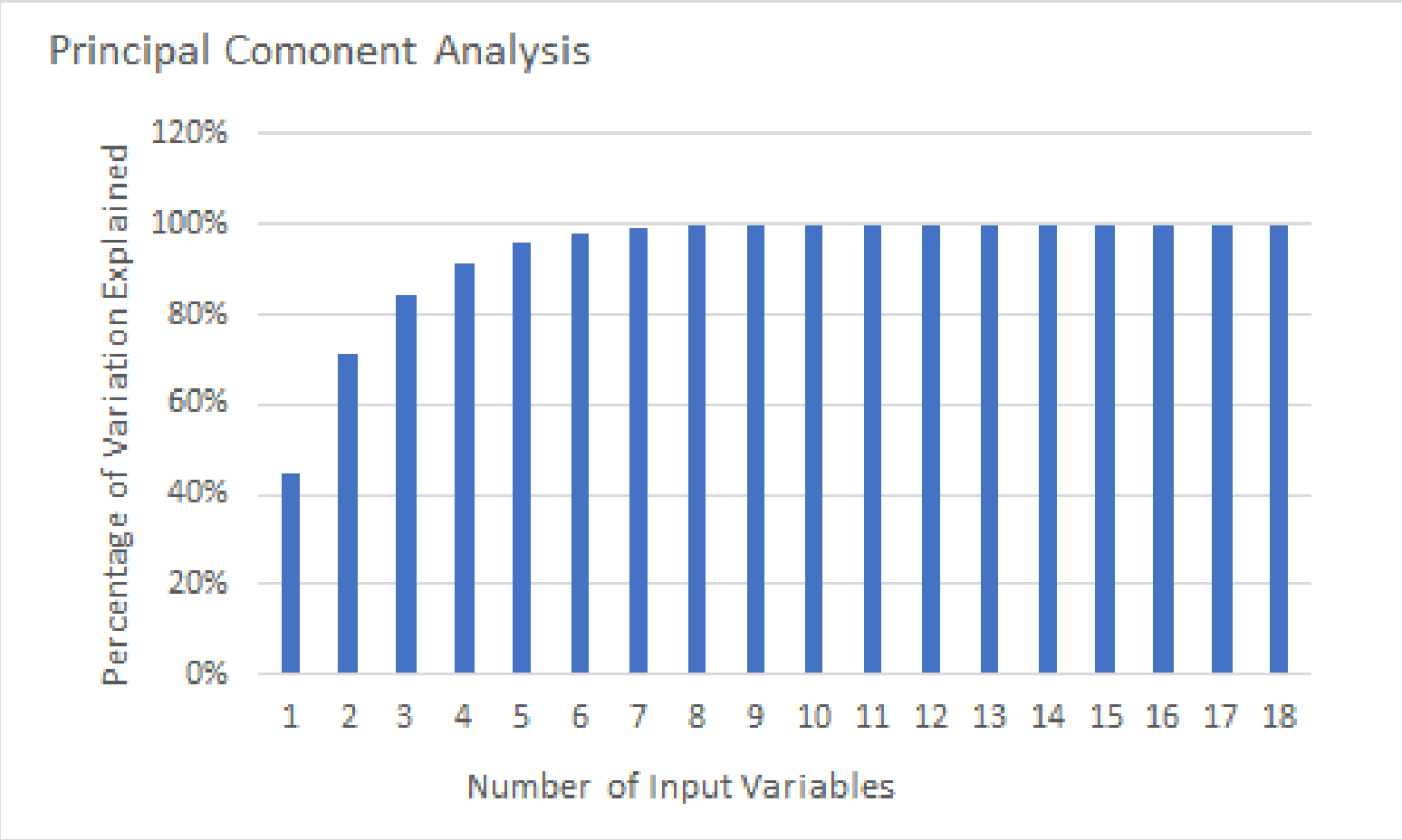
- Risk Reduction
- RC
- MCR
- Excess Size
- Excess Volatility
- Excess Weight
- Excess Price
- SS POV
- Number Stocks = 1/N

Output Data:

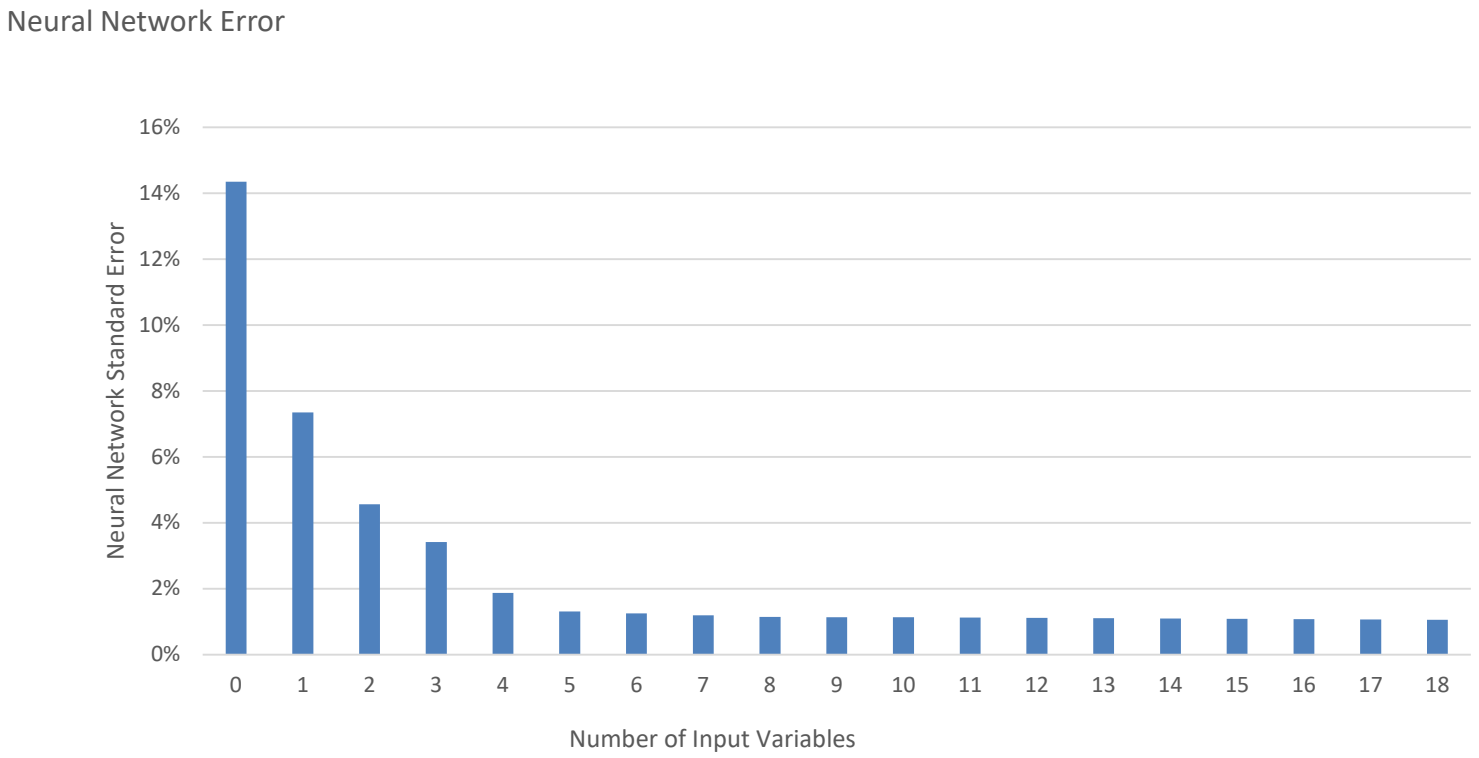
Y = Optimal POV Rate

(Determined from Optimization Routine)

Principal Component Analysis



Best Fit NNET Model



Results

Best Predictive Variables

Input Data:

1. Lambda
2. IStar
3. Timing Risk
4. Risk Change
5. Risk Contribution
6. Excess Size
7. Excess Volatility
8. Excess Weight

NNET Structure:

The best NNET performance consisted of eight (8) input factors and three hidden layers.

- First Hidden Layer = 4 nodes and Inverse Tangent function.
- Second Hidden Layer = 8 nodes and Inverse Tangent function.
- Third Hidden Layer = 4 nodes and Sigmoid function.

Best Fit NNET Model

Comparison of NNET Initial Trade Rate Solution to Optimal Trade Rate Solution

Experiment (Part 2): Calculating Solution Time

Calculating Solution Time Improvement

Step I:

For each of the 2,100 baskets we solved the Multi-Period Trade Schedule Optimization Problem using a random initial starting value.

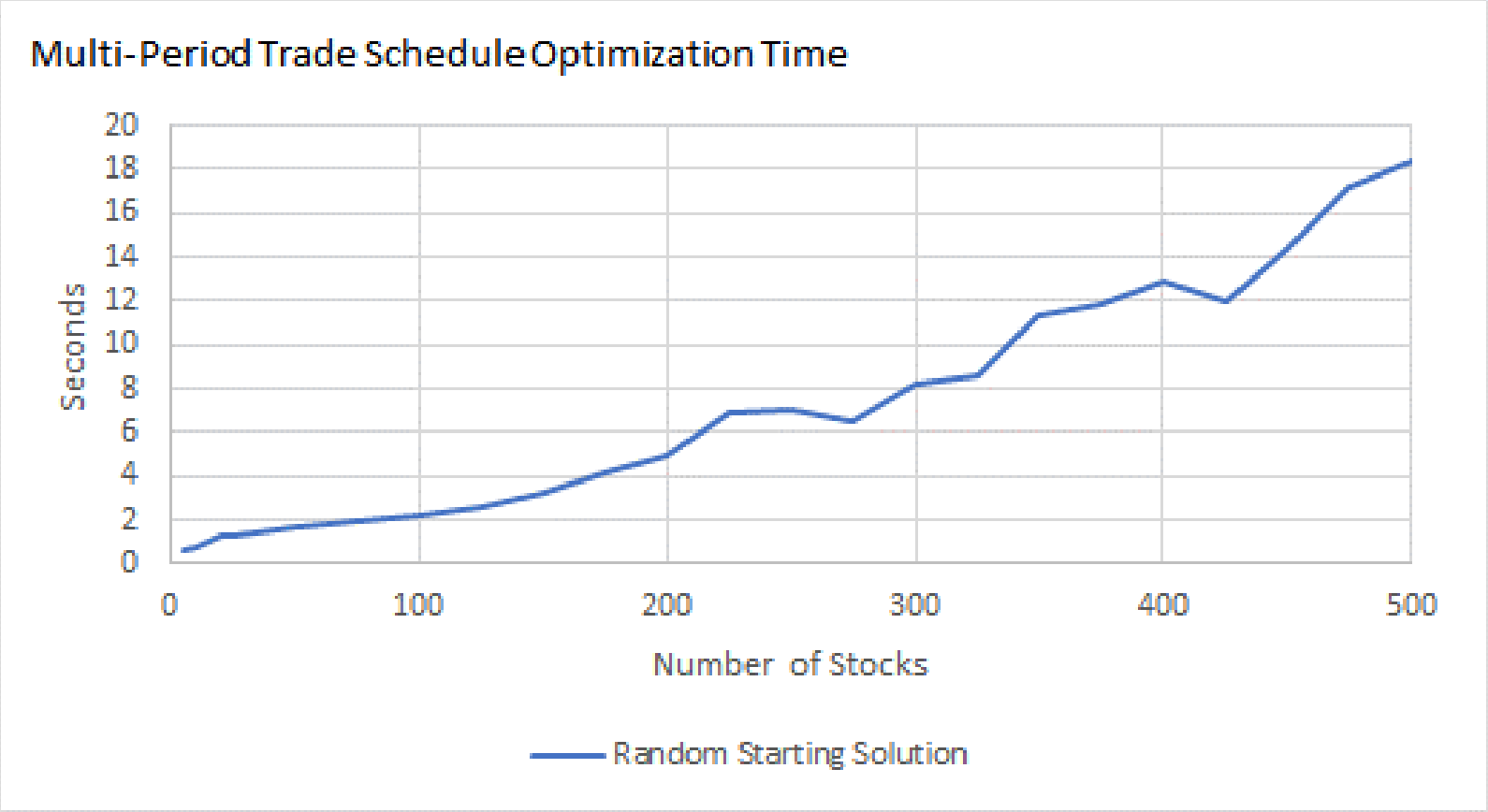
- We recorded the calculation Time

Step II:

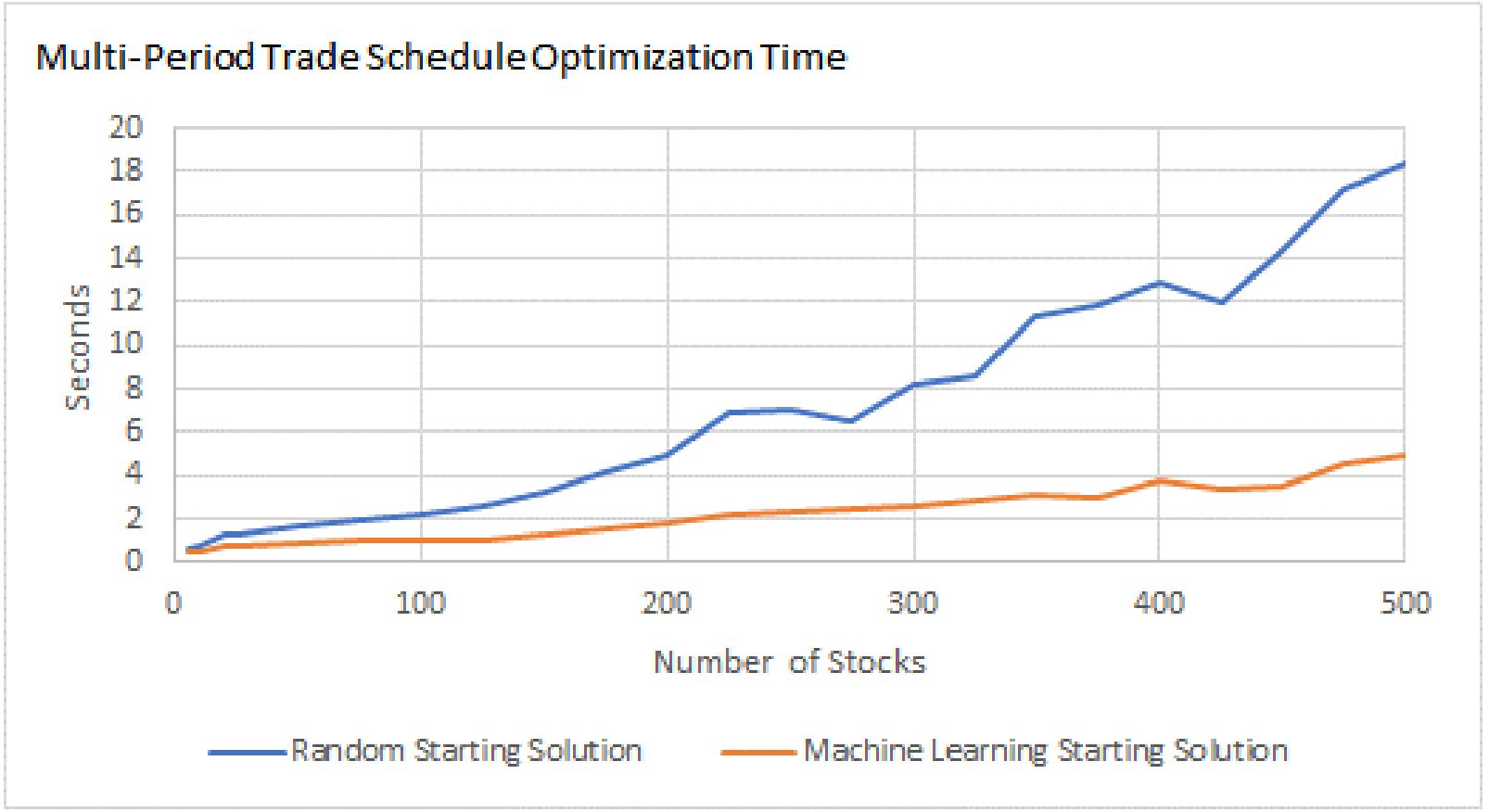
For each of the 2,100 baskets, we solved the Multi-Period Trade Schedule Optimization Problem using the NNET solution as the initial starting value.

- We recorded the calculation Time.

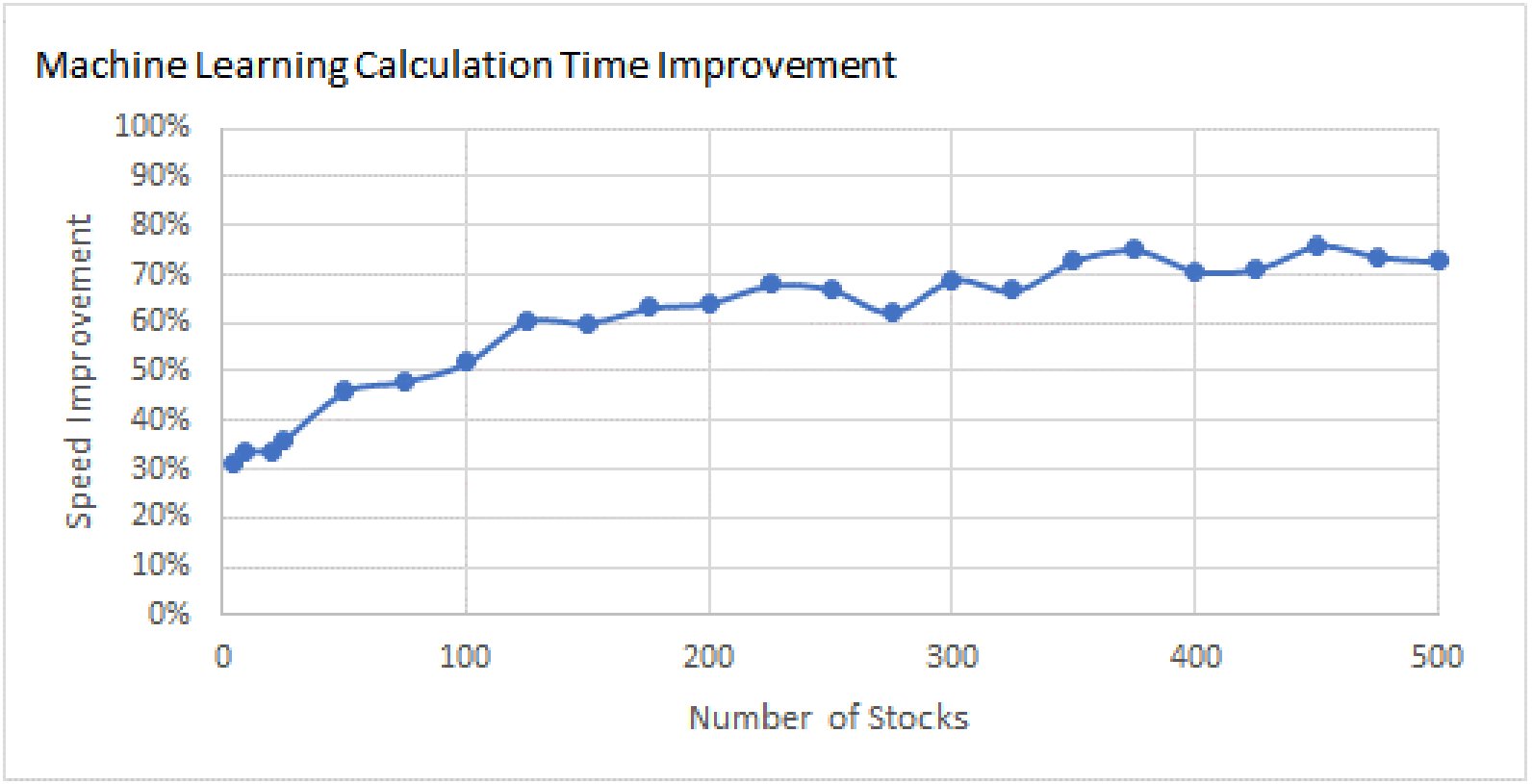
Trade Schedule Optimization – Calculation Time



Trade Schedule Optimization – Calculation Time



Best Fit NNET Model



Conclusions

Conclusions

- Combining Machine Learning (NNET) with Multi-Period Trade Schedule Optimization to determine the starting solution provided an improvement in calculation speed of:
  - 30% faster for portfolios with n=10 stocks
  - 50% faster for portfolios with n=100 stocks
  - 70% faster for large portfolios (e.g., 300-500 stocks).

- These improvements in calculation time will allow investors to react to profitably market conditions and take advantage of favorable market conditions when they arise in the marketplace.

Questions ???

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