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Why a Total Portfolio Risk Model is not the Sum of the Specialist Models

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A Brief History of Multi-Asset Class Risk Models

- *In the Beginning There Were the Fundamental Investors…* They were looking to buy securities in an orderly market
- Dealers and market makers obliged by carrying inventories of securities in demand
- Market makers made money on the bid-ask spread, not by betting on market movements, which they disliked
- They devised "volatility models" to estimate what they can loose at a certain confidence level, and hence adjust the bid-ask spread they charge
- To utilize economies of scale, market maker firms ran several different inventories
- Simple covariances between the different security inventories helped estimate how much the firm stood to loose at a certain confidence level in the market outcomes

A Brief History (cont'd)

- Bill Sharpe's CAPM, Harry Markowitz' MVO, and Richard Ross' APT emerged. Buy-side investors started considering "portfolio efficient" investing which was not just about standalone company characteristics.

- It looked like the dealer risk systems were speaking similar language to MPT, so they were considered a natural start.

- The first buy-side risk systems emerged. To make their systems marketable, vendors tried to make them similar to the "inventory by inventory" approach of dealers. The consideration was that if they were too different and give too different results for each individual security compared to the dealer estimates, investors will not trust them.

A Brief History (cont'd)

- As investors adopted ever increasing level of diversification, the risk systems readily accommodated that by adding more dimensions of "risk factors". E.g. a credit spread factor for Spanish BB rated bonds, a factor option implied volatility of a specific issuer, a specific tenor of a interest rate swap in a specific currency, etc.

- Even with more than a *thousand risk factors*, it was claimed that the precision of modeling movements at the individual security level meant precision at the portfolio level.

- The fact that the observations on each separate risk factor dimension was kept relatively constant, seemed of secondary importance.

- We will show why the large number of risk factors combined with a limited number of historical observations is a practice that is *designed* to introduce randomness in the result beyond the uncertainty in the risk factors

In a world with stable betas, no idiosyncratic risk…

- All investment returns will be explained by the risk factors
- In this case, we do not need any statistical or econometric technique, just some linear algebra

$$\begin{matrix} F_{1,t1} & F_{2,t1} & F_{3,t1} \\ F_{1,t2} & F_{2,t2} & F_{3,t2} \\ F_{1,t3} & F_{2,t3} & F_{3,t3} \end{matrix} \cdot \begin{matrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{matrix} = \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$$

- F is the historical factor realization of the particular risk factor, R is the return of the specific investment, and β is the factor exposure of the investment to the risk factor.

- We can solve the above identity for the factor exposures by multiplying both sides by the inverse of the matrix of the historical factor returns

What if we looked at a *longer* historical period

- All investment returns will still be explained by the risk factors
- In this case, we still do not need any statistical or econometric technique, just some linear algebra. The following will still hold true.

$$\begin{matrix} F_{1,t1} & F_{2,t1} & F_{3,t1} \\ F_{1,t2} & F_{2,t2} & F_{3,t2} \\ F_{1,t3} & F_{2,t3} & F_{3,t3} \\ F_{1,t4} & F_{2,t4} & F_{3,t4} \\ F_{1,t5} & F_{2,t5} & F_{3,t5} \end{matrix} \cdot \begin{matrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{matrix} = \begin{matrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_3 \end{matrix}$$

- The problem is "over-specified", but still solvable. If there are K factors, take any K historical periods and form K by K historical factor matrix, and then proceed to solve for the factor betas in the usual fashion to solve a system of K equations with K variables described previously.

What if we looked at a *shorter* historical period

- All investment returns will still be explained by the risk factors
- Our starting point is to still look at the matrix identity:

$$\begin{matrix} F_{1,t1} & F_{2,t1} & F_{3,t1} \\ F_{1,t2} & F_{2,t2} & F_{3,t2} \end{matrix} \cdot \begin{matrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{matrix} = \begin{matrix} R_1 \\ R_2 \end{matrix}$$

- This problem has an infinite number of solutions
- Seems like an impasse
- The acknowledgement of idiosyncratic returns comes to the “rescue”

A Magic Carpet called "Idiosyncratic Returns"

- In a fully specified world that includes idiosyncratic returns $\boldsymbol{\varepsilon}$:

$$\begin{matrix} F_{1,t1} & F_{2,t1} & F_{3,t1} \\ F_{1,t2} & F_{2,t2} & F_{3,t2} \\ F_{1,t3} & F_{2,t3} & F_{3,t3} \end{matrix} \cdot \begin{matrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{matrix} + \begin{matrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{matrix} = \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$$

- In an underspecified world we have the following… not much help

$$\begin{matrix} F_{1,t1} & F_{2,t1} & F_{3,t1} \\ F_{1,t2} & F_{2,t2} & F_{3,t2} \end{matrix} \cdot \begin{matrix} \beta_1 \\ \beta_2 \end{matrix} + \begin{matrix} \varepsilon_1 \\ \varepsilon_2 \end{matrix} = \begin{matrix} R_1 \\ R_2 \end{matrix}$$

- We then remember that there are tools in econometrics (regressions, PCA, factor analysis) that can help "solve" even the underspecified case

A Carpet called "Idiosyncratic Returns" (cont'd)

- Econometrics seems like a rational choice:
  - How else would we estimate idiosyncratic returns
  - By definition, they are unobservable and exist only in contrast to factor driven returns
  - Econometrics is the only way to empirically separate factor driven returns from idiosyncratic returns

- Econometrics can be a blunt force instrument: it does not explicitly care if the original problem with the factor betas is over- or underspecified

- Most often than not it is trying to minimize the sum of the squared residuals
  - $argmin_{\boldsymbol{\beta}} \sum_{i=1}^{N} (\sum_{j=1}^{K} F_{i,j} * \beta_j - R_i)^2 = \sum_{i=1}^{N} \varepsilon^2$

A Carpet called "Idiosyncratic Returns" (cont'd)

- Let us take two separate two separate sub-samples of history:

$$\begin{matrix} F_{1,t1} & F_{2,t1} & F_{3,t1} \\ F_{1,t2} & F_{2,t2} & F_{3,t2} \end{matrix} \cdot \begin{matrix} \beta_1 \\ \beta_2 \end{matrix} + \begin{matrix} \varepsilon_1 \\ \varepsilon_2 \end{matrix} = \begin{matrix} R_1 \\ R_2 \end{matrix}$$

$$\begin{matrix} F_{1,t1} & F_{2,t1} & F_{3,t1} \\ F_{1,t5} & F_{2,t5} & F_{3,t5} \end{matrix} \cdot \begin{matrix} \beta_1 \\ \beta_2 \end{matrix} + \begin{matrix} \varepsilon_1 \\ \varepsilon_5 \end{matrix} = \begin{matrix} R_1 \\ R_5 \end{matrix}$$

- If we use multiple linear regression, the solution for the factor exposure vectors is

$$\boldsymbol{\beta} = (\boldsymbol{F'} * \boldsymbol{F})^{-1} * \boldsymbol{F'} * \boldsymbol{R}$$

The $(\boldsymbol{F'} * \boldsymbol{F})$ matrix however is not invertible when N < K, hence a solution for exposures can't be found in this way (already a red flag)

- We can then consider using step-wise regression which does not suffer from this problem. However…

A Carpet called "Idiosyncratic Returns" (cont'd)

- If we have two different set of factor returns in the second and onward stages of the step-wise regression, we will get different betas.

$$\begin{matrix} F_{1,t1} & F_{2,t1} & F_{3,t1} \\ F_{1,t2} & F_{2,t2} & F_{3,t2} \end{matrix} \cdot \begin{matrix} \beta_1 \\ \beta_2 \end{matrix} + \begin{matrix} \varepsilon_1 \\ \varepsilon_2 \end{matrix} = \begin{matrix} R_1 \\ R_2 \end{matrix}$$

$$\begin{matrix} F_{1,t1} & F_{2,t1} & F_{3,t1} \\ F_{1,t5} & F_{2,t5} & F_{3,t5} \end{matrix} \cdot \begin{matrix} \beta_1 \\ \beta_2 \end{matrix} + \begin{matrix} \varepsilon_1 \\ \varepsilon_5 \end{matrix} = \begin{matrix} R_1 \\ R_5 \end{matrix}$$

- By taking sub-samples we have introduced, by design, randomness in the factor exposures. *The stated necessity that led to do this outcome was "the need to estimate idiosyncratic return econometrically".*

- As we expand the portfolio, and with that the number of factors, errors become numerous. They are not independent neither across factors nor across securities.

The Factor Covariance Matrix

- The observations from the prior discussion point to two major problems for the enterprise risk model designed in this fashion:
  - The factor betas get systematically biased as the portfolio includes more securities
  - The correlation between factors themselves introduce estimation errors. One additional factor introduces K - 1 errors with the other factors, which do not diversify across the portfolio, due to the fact that factors are pervasive across securities.

- This outcome is a direct consequence of the decision to “be precise factor-wise at the individual security level”, and the consequent increase of the number of factors roughly with the increase of the number of security types included in the portfolio.

Alternatives to Address The Problem:

1. **Plead Ignorance:** Pretend the problem does not exist and report risk in this fashion.

2. **Play coy:** Associate the errors in the covariance matrix to the sidecar of "model risk ". Apart from that do nothing else.

3. **Attempt Valiantly.** Apply various shrinkage techniques on the factor betas and the factor covariance matrix. That will tend to make the estimates more stable but should be used with caution because:

   a) may introduce bias

   b) does not address the problem that a thousand factor bets are difficult to trace, interpret, and act upon

4. **Build a Parsimonious Model.** The solution that will address all total fund risk management concerns, as we shall see...

A Parsimonious Model

1. Has a relatively narrow set of factors. Thus:
   - does not create challenges in the stability of the factor exposures
   - does not introduce large number of estimation errors to the factor covariance matrix

2. Should have enough factors to explain *common* movements across assets, and no more factors than that.

3. A Sample Parsimonious Model Factors for all Asset Classes, Globally

   - *Sector index performance*
   - *Region index performance*
   - *Exogenous costs changes (energy, borrowing)*
   - *Style indices*
   - *Currency factors*
   - *TVM and Inflationary Expectations - Yield curve change factors*

However, do we see a Discrepancy ?

1. In practice, some have raised a concern about the communication between the total fund risk management and the specialist mandates.
   - If total fund risk management is using a different risk model than the one used by the specialist, how are risk bets of the specialist measured in the context of the overall portfolio?
   - If the risk of the particular mandate in the enterprise context is too large or small, how does total fund management communicate particular dimensions of risk adjustment of the specialist portfolio back to the particular mandate

2. Intuitively, this is not an insurmountable problem
   - We can express the specialist model risk factors as a function of the total fund risk model factors. Then the exposures to the specialist factors will multiply through to the enterprise risk model factors.
   - This is ok when we communicate specialist to the enterprise level, but also entails we have to do as many enterprise model translations as there are specialist models to communicate enterprise to specialist level

The Solution: Factor Mimicking Portfolios

- Factor mimicking portfolios (FMP) are portfolios that have an exposure of "1" to a specific factor F, and "0" to all other factors. They can be numerically solved form the exposures of the securities covered by any risk model.

- The specialist portfolio then is the linear combination of the FMPs of their risk model factors each weighted by its respective factor exposure.

- If we have a portfolio that behaves like a factor, then we can treat it as a composite asset in a larger portfolio where its weight is the exposure of that factor in the context of the specialist portfolio times the weight of the specialist portfolio in the context of the total portfolio.

Two Examples

- Northfield Fundamental Model

  Model Description: *Company characteristics like Beta, EPS Growth, EP, Revenue to Price, etc. are used as exposures to factors that cross-sectionally define, period after period, a set of factor returns that correspond to those labels, and, hence, measure the investor driven performance of stocks with those characteristics.*

- Northfield Real Estate Model

  Model Description: *Commercial Real Estate Properties are modelled as a combination of three components: "expected rent" cashflow stream that is treated as a long fixed income security with tenant credit risk, "financing" which is treated as a short fixed income security, and a "rent volatility" component which is treated similarly to an equity security. That is why the resulting factor exposures are those related to sectors, regions, and yield curve shape model factors.*

Fundamental Model Example: "Translate a Factor"

Factor Portfolio for Fundamental Risk Factor "Beta"					
Factor	**PortExp**	**BenchExp**	**ActiveExp**	**FactorVar**	**VarContr**
Beta	*0.997*	*1*	0	103.27	0
Earnings/Price	-0.006	0	-0.01	4.55	0
Book/Price	0.006	0	0.01	8.76	0
Dividend Yield	-0.007	0	-0.01	6.39	0
Trading Activity	0.001	0	0	4.97	0
Relative Strength	-0.005	0	0	25.67	0
Log of Market Cap	-0.005	0	-0.01	7.92	0
Earnings Variability	0.011	0	0.01	2.07	0
EPS Growth Rate	0.003	0	0	2.71	0
Revenue/Price	0.007	0	0.01	4.43	0
Debt/Equity	-0.003	0	0	0.94	0
Price Volatility	-0.008	0	-0.01	22.4	0
Major Banks	0.008	0	0.01	64.67	0
…	…	…	…	…	…

Impact of the Specialist Factor on the Total Portfolio

- We can demonstrate the use of FMP to measure the enterprise impact of a specialist risk factor using various decomposition approaches, but for now we show this can be done in a *X-Sigma-Rho breakdown* of the total portfolio's standard deviation.
- First calculate Rho - the correlation between the FMP and the Total Portfolio. The factor exposures of the FMP in the enterprise risk model are the linear combination of the factor exposures of the FMP constituents identified in the previous stage

$$Rho = Corr(FMP, TP) = \sum_{i=1}^{K} \sum_{j=1, j<>i}^{K} \beta_{j,FMP}\beta_{k,TP} COV(F_i, F_j) * \frac{1}{\sigma_{FMP} * \sigma_{TP}}$$

- We find that Rho = *0.63*
- X is the weight of the specialist portfolio as part of the total portfolio and it is 0.12. Sigma is the volatility of the FMP as scaled by the exposure of the specialist manager to that factor. It is 0.27. Contribution to risk of the total portfolio by the specialist beta factor bet = X * Sigma * Rho = 0.12 * 0.27 * 0.63 = 0.02 = 2%

Real Estate Model Example: "Translate an Asset"

- Similarly to the case of capturing the impact of the fundamental factor to the total fund risk, we can take all the factors in the Northfield Real Estate Risk Model and represent them with FMPs. Then we can solve for the linear combination of these FMPs that will reproduce for the factor exposures of the asset in the original risk model.

Factor Exposure	Property 13	Mimicking Portfolio
ASIA	0	8.67E-19
CONSUMER	0.109	0.109
ENERGY	0.000	0.000
ENGLISH	0.023	0.023
EUROPE	0	4.07E-19
INDUSTRIAL	0.008	0.008
INT RATE SECT	0.054	0.054
NON-ENERG MINERALS SECT	0.001	0.001
TRSRY CURVE F1	-23.508	-23.508
TRSRY CURVE F2	-219.997	-219.997
TRSRY CURVE F3	-2283.067	-2283.067
TECHNOLOGY	0.010	0.010

- The Asset Mimicking Portfolio is simply a combination of stocks and bonds identified by common CUSIPs, SEDOLs, and ISINs. The user can simply take the portfolio as a composite asset and thus incorporate the real estate asset in a third party risk model – e.g. Factset, Blackrock, MSCI, etc.

Summary

- We traced the evolution of multi-asset class risk models and the pitfalls that arose when linear thinking was extended outside of the limits of statistical stability
- We observed the mathematical origins of those pitfalls and explored the expected magnitude of their impact
- Most importantly, we concluded that the most robust solution for a total multi-asset class portfolio is a dedicated parsimonious risk model
- We have also demonstrated that there is a seamless ways to analyze the impact of any manager's proprietary risk factor within the context of the total portfolio risk model, without that factor being a part of the total portfolio risk model
- The collateral benefit of this approach is that we can include assets and asset types from one risk model to another risk model where they previously did not natively exist

Question and Answer Session

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